Chapter8 Inferences based on a single sample: Tests of Hypotheses

Besides estimating the value of population parameter, we are also interested in making an inference about how the value of a parameter relates to a _______________. Is it less than, equal to, or greater than the specific value?
This type of inference is called a _________________.

Determining the Target Parameter

Parameter	Key Words or Phrases	Type of Data
μ	Mean; average	Quantitative
p	Proportion; percentage; fraction; rate	Qualitative
σ^2	Variance; variability; spread	Quantitative

8.1 The elements of a test of hypothesis

A statistical hypothesis is a _________ about the numerical value of a population parameter.

Summary of elements of a test of hypothesis:

1. ______________ **(H_0):** represents the hypothesis that will be accepted unless the data provide convincing evidence that it is false. This generally represents the status quo, which we adopt until it is proven false.

H_0**:**________________________

__________________**(H_a):** represents the hypothesis that will be accepted only if the data provide convincing evidence of its truth.

H_a :

2. ________________**:** a sample statistic used to decide between the null and alternative hypothesis.

3. ________________**:** the set of possible values of the test statistic for which the researcher will reject _______________in favor of_________________. The rejection region is chosen so that the probability of rejecting a true null hypothesis is α (_______________).

4. ____________________**:**

a. If the numerical value of the test statistic ___________ **the rejection region**, we________ the **null hypothesis** and conclude that the alternative hypothesis is true.

b. If the numerical value of the test statistic__________________ **the rejection region**, we _______________ the **null hypothesis** and conclude that there is insufficient evidence to conclude that the alternative hypothesis is true.

- ## Basic logic of hypothesis testing:

1. Take a ________________ from the population of interest;
2. If the sample data __________ sufficient evidence to conclude that_________________is false, we reject H_0 and assert the alternative hypothesis.
3. If the sample data ______________ provide sufficient evidence to conclude that H_0 is false, we fail to reject H_0 and conclude there is not enough evidence to assert the alternative hypothesis.

- ## Significance level α

Conclusions and consequences for a test of hypothesis

conclusion	True state of Nature (H_0 in fact is)	

Court example: H_0: a person is innocent

Type I error:
Type II error:
Correct decision:
Correct decision:

Type I Error = _________ a null hypothesis when in fact it is ________

________= The probability of committing a Type I error

Type II Error = ___________ reject a null hypothesis when in fact it is ________

_________= The probability of committing a Type II error

___________ = $1-\beta$ = P(rejecting a null hypothesis when in fact it is false)

The relationship between α and β: If α is __________, then β will be ________; vice verse.

When we do the hypothesis test, ______will be controlled.

Typical values for α are: ____________________ .

Example: To investigate if the mean Statistics test score is greater than 75 in a college, $H_0: \mu = 75 \quad H_a: \mu > 75$

then, Type I Error = We ____________, when in fact the mean score is ____.($H_0: \mu = 75$ is true.)

Type II Error = We __________ , when in fact the mean score ______75 ($H_0: \mu = 75$ is false).

Note: when conclusion goes to ________ H_0, there will be only chance to make _____ error.

When conclusion goes to __________ H_0, there will be only chance to make _____ error.

- ## How to formulate the null hypothesis and alternative hypothesis:

Three points: 1. What is the target _________ (mean μ, proportion p or variance σ^2)?

2. What is the ______________ the parameter will be compared to?
3. What is the ____________ between the parameter and the specific value you are interested in comparing (not equal, less than or greater than)?

Examples: Set up null hypothesis and alternative hypothesis and specify Type I, II Error

Example1: SSHA: The Survey of Study Habits and Attitudes is a psychological test that measures the motivation, attitude, and study habits of college students. The mean score is expected 115. A survey based on 81 incoming freshmen result in mean score is 116.2 and standard deviation is 25. Do the data provide evidence that the mean SSHA score is different from 115?

Type I Error: we _____________, when in fact the mean SSHA score _____115 ($H_0: \mu = 115$ is __________.

Type II Error: we _____________, when in fact the mean SSHA score _____ 115. ($H_0: \mu = 115$ is __________.)

Example2. Gun Control: In a survey conducted by Louis Harris of LH research, 1250 U.S. adults were polled regarding their views on banning handgun sales, 650 favored a ban.
At 5% significance level, do the data provide sufficient evidence to conclude that more than 50% of U.S. adults favor banning handgun sales?

Type I Error: we _______ , when in fact the proportion of favoring banning gun sales ____ 0.5.
Type II Error: we_______, when in fact the proportion of favoring banning gun sales ____ 0.5.

8.2 Large-sample test of hypothesis about one population mean

- Condition required for a valid large-sample hypothesis test for population mean μ :

1. A __________ sample is selected from the target population.
2. The sample size n is _________ (_______).

Under the conditions, by Central Limit Theorem,
The Sampling distribution of $\bar{x}$ is approximately normal with:

Mean: __________

Standard error: ______________

- <u>Large sample Z-test</u> of <u>hypothesis</u> for <u>a population mean</u> μ <u>:</u>

1. set up null hypothesis and alternative hypothesis

H_0 : ________________________________

H_a : $(two-tailed)$

$or\ H_a$: $(lower-tailed)$

$or\ H_a$: $(upper-tailed)$

2. significance level α

3. test statistic: $z = \dfrac{\bar{x}-\mu}{\sigma/\sqrt{n}} \approx$

4. rejection region: _____________when $H_a : \mu \neq \mu_0$

____________ when $H_a : \mu < \mu_0$

______________ when $H_a : \mu > \mu_0$

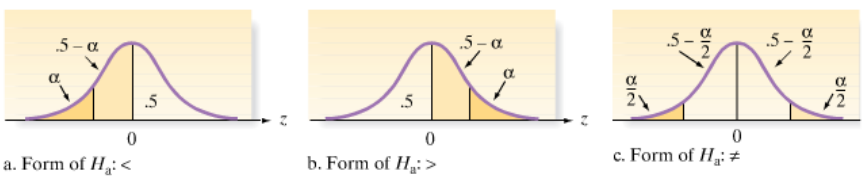


FIGURE 8.4
Rejection regions corresponding to one- and two-tailed tests

5. conclusion: if the value of test statistic **falls in** rejection region, ______ H_0 , and conclude that at α level, there is **sufficient** evidence to conclude H_a is true.

if the value of test statistic **does not fall in** rejection region, _____________ H_0, and conclude that at α level, there is **insufficient** evidence to conclude H_a is true.

Examples for Large-sample Z-test of hypothesis about a population mean μ

Problem The effect of drugs and alcohol on the nervous system has been the subject of considerable research. Suppose a research neurologist is testing the effect of a drug on response time by injecting 100 rats with a unit dose of the drug, subjecting each rat to a neurological stimulus, and recording its response time. The neurologist knows that the mean response time for rats not injected with the drug (the "control" mean) is 1.2 seconds. She wishes to test whether the mean response time for drug-injected rats differs from 1.2 seconds.

The sample of 100 drug-injected rats yielded mean response time is 1.05 seconds and standard deviation is 0.5 second, using $\alpha = 0.01$ to conduct this test.

At $\alpha = 0.01$, there is sufficient evidence to conclude the mean response time for drug-injected rats differs from the control mean of 1.2 second.

Step1: (State the null and alternative hypotheses mathematically and in words of the problem.)

Step2: (calculate the value of test statistic.)

Step3: (Find the rejection region.)

Step4: (Make a decision)

Step5: (state a complete conclusion.)

Example2, SSHA: The Survey of Study Habits and Attitudes is a psychological test that measures the motivation, attitude, and study habits of college students. The mean score is expected 115. A survey based on 81 incoming freshmen result in mean score is 116.2 and standard deviation is 25. Do the data provide evidence that the mean SSHA score is **higher from** 115? Use $\alpha = 0.05$.

Step1: (State the null and alternative hypotheses mathematically and in words of the problem.)

Step2: (calculate the value of test statistic.)

Step3: Find the rejection region

Step4: (Make a decision.)

Step5: (state a complete conclusion.)

Example3. HOSPLOS, The length of stay (in days) in hospital for 100 randomly selected hospital patients are presented in the table. Suppose we want to test the hypothesis that the true mean length of stay (LOS) at the hospital is **less than** 5 days. Use $\alpha = 0.05$.

LOS for 100 hospital patients	2, 3, 8, 6, 4, 4, 6, ………, 10, 2, 4, 2

Step1: (State the null and alternative hypotheses mathematically and in words of the problem.)

Step2: (calculate the value of test statistic.))

Step3: Find the rejection region

Step4: (Make a decision.)

Step5: (state a complete conclusion.)

SPSS output for HOSPLOS:

One-Sample Statistics

	N	Mean	Std. Deviation	Std. Error Mean
LOS	100	4.53	3.678	.368

One-Sample Test

	Test Value = 5					
					95% Confidence Interval of the Difference	
	t	df	Sig. (2-tailed)	Mean Difference	Lower	Upper
LOS	-1.278	99	.204	-.470	-1.20	.26

8.3 Observed significance levels: p-values

____________**(OSL):** the ___________ (assuming H_0 is true) of observing a value of the test statistic that is at least contradictory to the null hypothesis, and supportive of the alternative hypothesis, as the actual one computed from the sample data. This measure the disagreement with the null hypothesis based on the sample data.

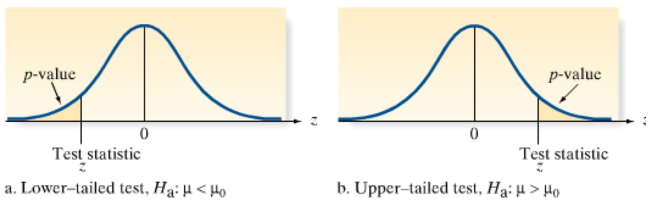
- How to calculate the p-value: (the value of test statistic and H_a)

1. Determine _______________________ based on the sample data;
2. **p-value** = ______________ when $H_a: \mu \neq \mu_0$

   p-value =______________ when $H_a: \mu < \mu_0$

   p-value =______________ when $H_a: \mu > \mu_0$

FIGURE 8.8
Finding the *p*-value for a one-tailed test



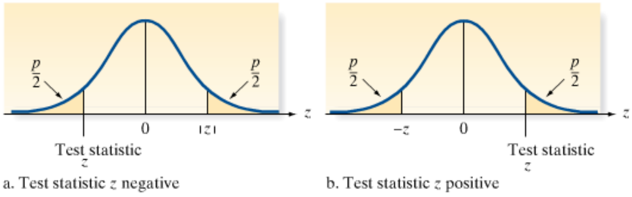


FIGURE 8.9
Finding the p-value for a two-tailed test: p-value $= 2(p/2)$

Examples for p-value calculation:

Recall three examples in section8.2 (using p-value to make decision)

Example1, The sample of 100 drug-injected rats yielded mean response time is 1.05 seconds and standard deviation is 0.5 second, using $\alpha = 0.01$ to test whether the mean response time for drug-injected rats differs from control mean of 1.2 second.

Step1: (State the null and alternative hypotheses mathematically and in words of the problem.)

Step2: (calculate the value of test statistic.))

Step3: Calculate p-value.

Step4: (Make a decision.)

Step5: (state a complete conclusion.)

Example2, SSHA: A survey based on 81 incoming freshmen result in mean SSHA score is 116.2 and standard deviation is 25. Do the data provide evidence that the mean SSHA score is **higher than 115**? Use $\alpha = 0.05$.

Step1: (State the null and alternative hypotheses mathematically and in words of the problem.)

Step2: (calculate the value of test statistic.))

Step3: Calculate p-value.

Step4: (Make a decision.)

Step5: (state a complete conclusion.)

Example3. HOSPLOS, The length of stay (in days) in hospital for 100 randomly selected hospital patients are presented in the table. Suppose we want to test the hypothesis that the true mean length of stay (LOS) at the hospital is **less than** 5 days. Use $\alpha = 0.05$.

LOS for 100 hospital patients	2, 3, 8, 6, 4, 4, 6, ………, 10, 2, 4, 2

Step1: (State the null and alternative hypotheses mathematically and in words of the problem.)

Step2: (calculate the value of test statistic.))

Step3: Calculate p-value.

Step4: (Make a decision.)

Step5: (state a complete conclusion.)

SPSS output for HOSPLOS,

One-Sample Statistics

	N	Mean	Std. Deviation	Std. Error Mean
LOS	100	4.53	3.678	.368

One-Sample Test

	Test Value = 5					
					95% Confidence Interval of the Difference	
	t	df	Sig. (2-tailed)	Mean Difference	Lower	Upper
LOS	-1.278	99	.204	-.470	-1.20	.26

Note: compare the examples in sec8.2 and sec8.3, you can find out either using rejection region or p-value approach to make decision always leads to the same conclusion.

- Two approaches to do hypothesis test:

A. _______________ Approach:
1. set up null hypothesis and alternative hypothesis,
2. calculate test statistic,
3. find appropriate rejection region,
4. make conclusion. (if test statistic falls in R. R, reject null hypothesis.)

B. ________________ Approach:
1. set up null hypothesis and alternative hypothesis,
2. calculate test statistic,
3. find p-value for the test,
4. make conclusion. (if p-value is smaller than α, reject null hypothesis.)

8.4 Small-Sample test of hypothesis about a population mean

- **Condition required for a valid small-sample hypothesis test for μ:**

1. A ___________________is selected from the target population;
2. The population from which the sample is selected has a distribution that is approximately______________.
3. Sample size is small (   ).

- Small sample t- test of hypothesis for a population mean μ :

1. set up null hypothesis and alternative hypothesis

$$H_0: \mu = \mu_0$$

$$\begin{aligned} & H_a: \mu \neq \mu_0 \quad (two-tailed) \\ or\ & H_a: \mu < \mu_0 \quad (lower-tailed) \\ or\ & H_a: \mu > \mu_0 \quad (upper-tailed) \end{aligned}$$

2. significance level α

3. test statistic: $t =$ with df = ________

4. rejection region: ___________when $H_a: \mu \neq \mu_0$

__________ when $H_a : \mu < \mu_0$

___________when $H_a : \mu > \mu_0$

5. conclusion: if the value of test statistic falls in R.R, reject H_0, and conclude that at α level, there is sufficient evidence to conclude H_a is true.

if the value of test statistic does not fall in R.R, do not reject H_0, and conclude that at α level, there is insufficient evidence to conclude H_a is true.

- How to find the p-value: (the value of test statistic and H_a**)**

1. Determine **the value of test statistic**__________________ based on the sample data;
2. **p-value** = _____________ when $H_a : \mu \neq \mu_0$

   p-value =______________ when $H_a : \mu < \mu_0$

   p-value =______________ when $H_a : \mu > \mu_0$

Note: since t-table in our textbook only provides some critical t-values, we can't use t-table to find exact p-value. We can use statistical software to calculate the appropriate p-value.

Examples for <u>Small-sample</u> t-test of hypothesis about a population mean μ

Example1. EMISSION,

A major car manufacturer wants to test a new engine to determine whether it meets new air-pollution standards. The mean emission of this type engines must be **less than 20** parts per million of carbon. Ten engines are manufactured for testing purposes. The data is listed below.

Emissions	15.6	16.2	22.5	20.5	16.4	19.4	19.6	17.9	12.7	14.9

Do the data supply sufficient evidence to allow the manufacturer to conclude that the type of engine **meets the pollution standard**? Assume to risk a type I error with probability α = 0.01.

Step1: (State the null and alternative hypotheses mathematically and in words of the problem.)

Step2: (calculate the value of test statistic.)

Step3: (Find the rejection region.)

Step4: (Make a decision)

Step5: (state a complete conclusion.)

SPSS output for EMISSION,

One-Sample Statistics				
	N	**Mean**	**Std. Deviation**	**Std. Error Mean**
Emission	10	17.5700	2.95223	.93358

One-Sample Test						
	Test Value = 20					
	t	**df**	**Sig. (2-tailed)**	**Mean Difference**	**95% Confidence Interval of the Difference**	
					Lower	**Upper**
Emission	-2.603	9	.029	-2.43000	-4.5419	-.3181

Example2,

Most water-treatment facilities monitor the quality of their drinking water on an hourly basis. One variable monitored is pH, which measures the degree of alkalinity or acidity in the water. A pH below 7.0 is acidic, one above 7.0 is alkaline, and a pH of 7.0 is neutral. One water-treatment plant has a target pH of 8.5. (Most try to maintain a slightly alkaline level.) The mean and standard deviation of 1 hour's test results, based on 17 water samples at this plant, are

$$\bar{x} = 8.42 \qquad s = .16$$

Does this sample provide sufficient evidence that the mean pH level in the water differs from 8.5?

Step1: (State the null and alternative hypotheses mathematically and in words of the problem.)

Step2: (calculate the value of test statistic.)

Step3: (Find the rejection region.)

Step4: (Make a decision)

Step5: (state a complete conclusion.)

8.5 Large-sample test of hypotheses about a population proportion

- **Condition required for a valid large-sample hypothesis test for p :**

1. A______________ sample is elected from a binomial population;
2. The sample size n is ______(This condition will be satisfied if _________________________)

- Large-sample test for a population proportion p :

1. set up null hypothesis and alternative hypothesis

H_0 :

H_a : $(two-tailed)$

or H_a : $(lower-tailed)$

or H_a : $(upper-tailed)$

2. significance level α

3. test statistic: $z=$

4. rejection region: ________________when $H_a: p \neq p_0$

_______________ when $H_a: p < p_0$

________________when $H_a: p > p_0$

5. conclusion: if the value of test statistic falls in R.R, reject H_0, and conclude that at α level, there is sufficient evidence to conclude H_a is true.

if the value of test statistic does not fall in R.R, do not reject H_0, and conclude that at α level, there is insufficient evidence to conclude H_a is true.

Examples for Large-sample test for a population proportion p :

Example1. Shipment Defectives,

The reputations of many businesses can be severely damaged by shipments of manufactured items that contain a large percentage of defectives. A manufacturer of alkaline batteries wants to be reasonably certain that fewer than 5% of its batteries are defective. Suppose 300 batteries are randomly selected from a large shipment, 10 defective batteries are found.

Does this provide sufficient evidence for the manufacturer to conclude that **the fraction** defective in the entire shipment is **less than 0.05**? Use $\alpha = 0.01$.

Step1: (State the null and alternative hypotheses mathematically and in words of the problem.)

Step2: (calculate the value of test statistic.)

Step3: (Find the rejection region.)

Step4: (Make a decision)

Step5: (state a complete conclusion.)

Example2. Gun Control: In a survey conducted by Louis Harris of LH research, 1250 U.S. adults were polled regarding their views on banning handgun sales, 650 favored a ban.

At 10% significance level, do the data provide sufficient evidence to conclude that **more than half** of U.S. adults favor banning handgun sales?

Step1: (State the null and alternative hypotheses mathematically and in words of the problem.)

Step2: (calculate the value of test statistic.)

Step3: (Find the rejection region.)

Step4: (Make a decision)

Step5: (state a complete conclusion.)

Learning objective of Chapter 8:

1. describe the basic elements of a test of hypothesis
2. Understand type I, type II error; significance level α; set up H_0, H_a; specify rejection region.
3. conduct a compete z-test of population mean (5 steps, critical value approach)
4. conduct a compete t-test of population mean (5 steps, critical value approach)
5. conduct a compete z-test of population proportion (5 steps, critical value approach)
6. know how to use p-value to make decision (if p-value < α, reject H_0)