Chapter 10 Analysis of variance (ANOVA)

10.1 Elements of a designed experiment

1. The ________________**(dependent variable)** is the variable of interest to be **measured** in the experiment.

2. ____________ are those variables whose **effect** on the response is of interest to the experimenter.

3. _______________are the values of the factor utilized in the experiment.

4. The_________________ are the **factor-level combinations** utilized.

5. An _________________is the **object** on which the response and factors are observed or measured.

6. A _______________________is one for which the analyst **control** the specification of the treatments and the method of assigning the experimental units to each treatment.

7. A _________________________is one for which the analyst simply **observes** the treatments and the response on a sample of experimental units.

Example 1: Extinct New Zealand birds. Refer to the evolutionary ecology research (July 2003) study of extinction in the New Zealand bird population. Ex1.18. Recall that biologists measured the body mass (in gram) and habitat types (aquatic, ground terrestrial, or aerial terrestrial) for each bird species. One objective is to compare the body mass means of birds with the three different habitat types.

1. response variable:______________________;
2. experiment unit:___________;
3. factor: _________________________;
4. treatment:____.

Example 2: An experiment was conducted to explore the effect of vitamin B12 on the weight gain of pigs. 40 similar piglets were randomly divided into 5 groups. 5 levels of vitamin B12 (0, 5, 10, 15, and 20 mgs./pound of corn meal ration) were randomly assigned to the 5 groups. At the end of the exp. period, each piglet weight gain in pounds was recorded.

1. response variable:___________________,
2. experiment unit:_____________;
3. factor:________________________;
4. treatment: ___________________________________.

Example 3: What are the treatments for a designed experiment with **two factors**, one qualitative with two levels (A and B) and one quantitative with five levels(50, 60, 70, 80, and 90)?
Treatments (Factor-level combinations):

10.2 The completely randomized design (CRD)

Two ways lead to a completely randomized design:

1. randomly assign ______________to the______________________;
2. randomly select ______________________of E.U.s for each___________________.

Data layout: (one-way table)

Treatment(k)			
1	2	……..	k

Objective of a CRD:____.

$$H_0 : \mu_1 = \mu_2 = ... = \mu_k$$ (____)

$$H_a : \mu_i \neq \mu_j \text{ for some } i \neq j$$ (____)

Test statistic?

Basic idea for testing: If the differences between ____________________are significantly greater than the amount of______________________, then we conclude at least two treatment means are different.

- **The variation between the treatment means is measured by:**

________________________________:

CM = _____________________

- **The total variation is measured by:**

________________________________:

$\sum x_i^2$ **: sum of observation squares (square each observation, then sum up)**

- **The variation within the treatment is measured by:**

________________________________:

Note: __________________________________

Degree of freedom: ______________________________________

To make the two measurements of variability comparable, we use

mean square for treatment : ____________

mean square for error: ________________

Test statistic:________________,

Rejection region:_________________.

Conclusion.

*** Critical value:** F_{α} with ____________ degree of freedom in the numerator (MST) and ________________degree of freedom in the denominator (MSE). (p799-p806, Table VIII--XI)

- **The properties of F-distribution:**

1.________skewed;
2. the area under the curve equals _____.
3. associated with two degrees of freedom.

Find some F-critical values.

Summary ANOVA table for CRD:

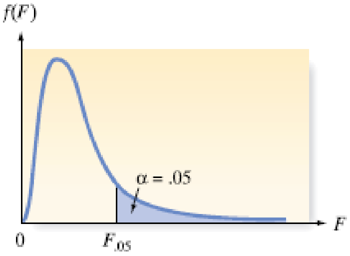
Source	df	SS	MS	F

- Conditions required for a valid ANOVA F-test: CRD

1. __________________________for each treatment;
2. All k sampled populations have approx. __________distributions;
3. The k population variances are_______________.

If the assumptions are not satisfied, use nonparametric methods.

TABLE IX Percentage Points of the F-distribution, $\alpha = .05$



NUMERATOR DEGREES OF FREEDOM

DENOMINATOR DEGREES OF FREEDOM

ν_2 \ ν_1	1	2	3	4	5	6	7	8	9
1	161.4	199.5	215.7	224.6	230.2	234.0	236.8	238.9	240.5
2	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28
26	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.77
27	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.25
28	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24
29	4.18	3.33	2.93	2.70	2.55	2.43	2.35	2.28	2.22
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12
60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04
120	3.92	3.07	2.68	2.45	2.29	2.17	2.09	2.02	1.96
∞	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88

Source: From M. Merrington and C. M. Thompson, "Tables of Percentage Points of the Inverted Beta (F)-Distribution," *Biometrika,* 1943, 33, 73–88.

(continued)

TABLE IX Continued

ν_2 \ ν_1 (DENOMINATOR DEGREES OF FREEDOM)	NUMERATOR DEGREES OF FREEDOM									
	10	**12**	**15**	**20**	**24**	**30**	**40**	**60**	**120**	**∞**
1	241.9	243.9	245.9	248.0	249.1	250.1	251.1	252.2	253.3	254.3
2	19.40	19.41	19.43	19.45	19.45	19.46	19.47	19.48	19.49	19.50
3	8.79	8.74	8.70	8.66	8.64	8.62	8.59	8.57	8.55	8.53
4	5.96	5.91	5.86	5.80	5.77	5.75	5.72	5.69	5.66	5.63
5	4.74	4.68	4.62	4.56	4.53	4.50	4.46	4.43	4.40	4.36
6	4.06	4.00	3.94	3.87	3.84	3.81	3.77	3.74	3.70	3.67
7	3.64	3.57	3.51	3.44	3.41	3.38	3.34	3.30	3.27	3.23
8	3.35	3.28	3.22	3.15	3.12	3.08	3.04	3.01	2.97	2.93
9	3.14	3.07	3.01	2.94	2.90	2.86	2.83	2.79	2.75	2.71
10	2.98	2.91	2.85	2.77	2.74	2.70	2.66	2.62	2.58	2.54
11	2.85	2.79	2.72	2.65	2.61	2.57	2.53	2.49	2.45	2.40
12	2.75	2.69	2.62	2.54	2.51	2.47	2.43	2.38	2.34	2.30
13	2.67	2.60	2.53	2.46	2.42	2.38	2.34	2.30	2.25	2.21
14	2.60	2.53	2.46	2.39	2.35	2.31	2.27	2.22	2.18	2.13
15	2.54	2.48	2.40	2.33	2.29	2.25	2.20	2.16	2.11	2.07
16	2.49	2.42	2.35	2.28	2.24	2.19	2.15	2.11	2.06	2.01
17	2.45	2.38	2.31	2.23	2.19	2.15	2.10	2.06	2.01	1.96
18	2.41	2.34	2.27	2.19	2.15	2.11	2.06	2.02	1.97	1.92
19	2.38	2.31	2.23	2.16	2.11	2.07	2.03	1.98	1.93	1.88
20	2.35	2.28	2.20	2.12	2.08	2.04	1.99	1.95	1.90	1.84
21	2.32	2.25	2.18	2.10	2.05	2.01	1.96	1.92	1.87	1.81
22	2.30	2.23	2.15	2.07	2.03	1.98	1.94	1.89	1.84	1.78
23	2.27	2.20	2.13	2.05	2.01	1.96	1.91	1.86	1.81	1.76
24	2.25	2.18	2.11	2.03	1.98	1.94	1.89	1.84	1.79	1.73
25	2.24	2.16	2.09	2.01	1.96	1.92	1.87	1.82	1.77	1.71
26	2.22	2.15	2.07	1.99	1.95	1.90	1.85	1.80	1.75	1.69
27	2.20	2.13	2.06	1.97	1.93	1.88	1.84	1.79	1.73	1.67
28	2.19	2.12	2.04	1.96	1.91	1.87	1.82	1.77	1.71	1.65
29	2.18	2.10	2.03	1.94	1.90	1.85	1.81	1.75	1.70	1.64
30	2.16	2.09	2.01	1.93	1.89	1.84	1.79	1.74	1.68	1.62
40	2.08	2.00	1.92	1.84	1.79	1.74	1.69	1.64	1.58	1.51
60	1.99	1.92	1.84	1.75	1.70	1.65	1.59	1.53	1.47	1.39
120	1.91	1.83	1.75	1.66	1.61	1.55	1.50	1.43	1.35	1.25
∞	1.83	1.75	1.67	1.57	1.52	1.46	1.39	1.32	1.22	1.00

ANOVA analysis procedure for completely randomized design (CRD):

1. Assumptions,
2. Summarize ANOVA table.
3. Conduct test of hypothesis to compare the k treatment means.

Examples of Completely Randomized Design:

Example 1, Teaching method: There were 15 students with similar IQ scores. Three different teaching methods were randomly assigned to these students, so for each method there were 5 students. After one year, these 15 students took the same test, the test scores were recorded. Do the data provide evidence that the mean test score are different among these three methods? Use $\alpha = 0.05$.

Method 1	Method 2	Method 3
78	68	65
81	93	79
69	94	90
82	87	81
75	83	75

E. U.:________________;
Treatments:_____________________; (k = ____)
Response variable: ________________(n = ____)

SPSS output for Teaching Methods:

Descriptives

test

	N	Mean	Std. Deviation	Std. Error	95% Confidence Interval for Mean		Minimum	Maximum
					Lower Bound	Upper Bound		
1	5	77.00	5.244	2.345	70.49	83.51	69	82
2	5	85.00	10.512	4.701	71.95	98.05	68	94
3	5	78.00	9.110	4.074	66.69	89.31	65	90
Total	15	80.00	8.759	2.261	75.15	84.85	65	94

ANOVA

test

	Sum of Squares	df	Mean Square	F	Sig.
Between Groups	190.000	2	95.000	1.290	.311
Within Groups	884.000	12	73.667		
Total	1074.000	14			

Note: The table below is also SPSS output which gives ANOVA information.

Tests of Between-Subjects Effects

Dependent Variable:score

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	190.000[a]	2	95.000	1.290	.311
Intercept	96000.000	1	96000.000	1303.167	.000
method	190.000	2	95.000	1.290	.311
Error	884.000	12	73.667		
Total	97074.000	15			
Corrected Total	1074.000	14			

a. R Squared = .177 (Adjusted R Squared = .040)

Example2, GOLFCRD,

USGA wants to compare the mean distances associated with four different brands of golf balls. A completely randomized design is used, with golfer Iron Byron, using a driver to hit a random sample of 10 balls of each brand in a random sequence. The distance is recorded for each hit.

Brand 1	Brand 2	Brand 3	Brand 4
251.2	263.2	269.7	251.6
245.1	262.9	263.2	248.6
248.0	265.0	277.5	249.4
251.1	254.5	267.4	242.0
260.5	264.3	270.5	246.5
250.0	257.0	265.5	251.3
253.9	262.8	270.7	261.8
244.6	264.4	272.9	249.0
254.6	260.6	275.6	247.1
248.8	255.9	266.5	245.9

Do the data provide evidence that the mean distances are different among these four brands? Use $\alpha = 0.10$.

E. U.: ____________________;
Treatments:______________________; (k = _____)
Response variable:_________________. (n = ______)

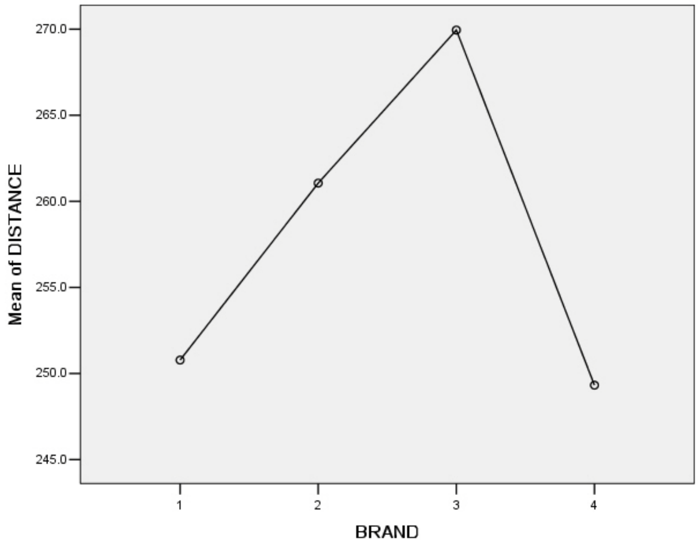
SPSS output for GOLFCRD:

Descriptives

DISTANCE

	N	Mean	Std. Deviation	Std. Error	95% Confidence Interval for Mean		Minimum	Maximum
					Lower Bound	Upper Bound		
1	10	250.780	4.7352	1.4974	247.393	254.167	244.6	260.5
2	10	261.060	3.8661	1.2226	258.294	263.826	254.5	265.0
3	10	269.950	4.5009	1.4233	266.730	273.170	263.2	277.5
4	10	249.320	5.2032	1.6454	245.598	253.042	242.0	261.8
Total	40	257.778	9.5497	1.5099	254.723	260.832	242.0	277.5

Mean plot



ANOVA DISTANCE					
	Sum of Squares	**df**	**Mean Square**	**F**	**Sig.**
Between Groups	2794.389	3	931.463	43.989	.000
Within Groups	762.301	36	21.175		
Total	3556.690	39			

Tests of Between-Subjects Effects

Dependent Variable: DISTANCE

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	2794.389(a)	3	931.463	43.989	.000
Intercept	2657969.580	1	2657969.580	125523.782	.000
BRAND	2794.389	3	931.463	43.989	.000
Error	762.301	36	21.175		
Total	2661526.270	40			
Corrected Total	3556.690	39			

a R Squared = .786 (Adjusted R Squared = .768)

Example 3, SCOPOLAMINE,

A completely random design was employed to examine the effect of drug scopolamine on memory. A total of 28 people were involved in this experiment. **Group 1**(12 people) were given an injection of scopolamine, **group 2(**8 people) were given an injection of placebo, **group 3(**8 people) were not given any drug. Four hours later, **the number of word pairs recalled** was recorded.

Group 1 (Scopolamine)	5, 8, 8, 6, 6, 6, 6, 8, 6, 4, 5, 6
Group 2 (Placebo)	8, 10, 12, 10, 9, 7, 9, 10
Group 3 (No drug)	8, 9, 11, 12, 11, 10, 12, 12

Is there evidence at $\alpha = 0.05$ to conclude that the mean number of word pairs recalled differs among the three treatment groups?

E. U.: _______________;
Treatments:____________________; (k = ______)
Response variable:__________________________, (n = ______)

SPSS output for Example 3, SCOPOLAMINE

Descriptives

RECALL

	N	Mean	Std. Deviation	Std. Error	95% Confidence Interval for Mean		Minimum	Maximum
					Lower Bound	Upper Bound		
1	12	**6.17**	1.267	.366	5.36	6.97	4	8
2	8	**9.38**	1.506	.532	8.12	10.63	7	12
3	8	**10.63**	1.506	.532	9.37	11.88	8	12
Total	28	8.36	2.407	.455	7.42	9.29	4	12

ANOVA

RECALL

	Sum of Squares	df	Mean Square	F	Sig.
Between Groups	**107.012**	**2**	**53.506**	**27.069**	**.000**
Within Groups	**49.417**	**25**	**1.977**		
Total	**156.429**	**27**			

Tests of Between-Subjects Effects

Dependent Variable:RECALL

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	107.012[a]	2	53.506	27.069	.000
Intercept	2054.083	1	2054.083	1039.165	.000
GROUP	107.012	2	53.506	27.069	.000
Error	49.417	25	1.977		
Total	2112.000	28			
Corrected Total	156.429	27			

a. R Squared = .684 (Adjusted R Squared = .659)

Exercise: Below is an incomplete ANOVA table for CRD.

Source	df	SS	MS	F
Diet	2			
Error		52.3		
Total	25	156.7		

1. Complete ANOVA table.
2. How many treatments are involved in this experiment?
3. How much is the MSE?
4. Write down the rejection region for hypothesis test of treatment means.

10.3 Multiple Comparisons of Means

- ___________________________________ can be used to find out which pair treatments are significantly different based on confidence interval of $(\mu_i - \mu_j)$, **when** ANOVA F-test for comparing treatment means leads to________________.
- **How many** pair-wise comparisons (μ_i, μ_j) of k treatment means:

____

if k = 3, c =____; if k = 4, c = ______

- **Three main procedures** are used to do multiple comparisons (ensure that the overall confidence level associated with all the comparisons remains at or above the specified 100(1-α)% level. α :__________________________rate)

Method	critical difference	treatment sample size	type of comparisons	C.I. of $(\mu_i - \mu_j)$
________	$w = q_{\alpha,(k,\upsilon)} \dfrac{s}{\sqrt{n_t}}$	_______	pairwise	
_________	$B_{ij} = t_{\alpha/(2c)} S \sqrt{\dfrac{1}{n_i} + \dfrac{1}{n_j}}$	________________	pairwise	
_________	$S_{ij} = \sqrt{(k-1)(F_\alpha)(MSE)(\dfrac{1}{n_i} + \dfrac{1}{n_j})}$	______________	general contrasts	

Note: **Scheffe's** C.I.s are ___________than the other two methods.

- **How to make decision:** If confidence interval of the difference $(\mu_i - \mu_j)$ ________________, it implies there is ______________________between the two treatment means.

  If confidence interval of the difference $(\mu_i - \mu_j)$ ___________________, it implies there is ________________________between the two treatment means.

- **How to express the result:**

1. put the treatment means in _________________________order.
2. put a bar over those pairs of treatment means which are **Not** significantly different. (Confidence interval for the difference ____________________________.)

Recall sec.10.2 Examples,

Example1, Teaching Method:

Method 1	Method 2	Method 3
78	68	65
81	93	79
69	94	90
82	87	81
75	83	75

Q: Do multiple comparisons of means of the teaching method data.

Questions: 1. Which teaching method(s) leads to the highest test score?

2. How many pair-wise comparisons of means (μ_i, μ_j) are there?

3. List those pairs of means which are sig. different.

4. List those pairs of means which are not sig. different.

Post Hoc Tests

Multiple Comparisons
Dependent Variable: test
Bonferroni

(I) method	(J) method	Mean Difference (I-J)	Std. Error	Sig.	95% Confidence Interval	
					Lower Bound	Upper Bound
1	2	-8.000	5.428	.499	-23.09	7.09
	3	-1.000	5.428	1.000	-16.09	14.09
2	1	8.000	5.428	.499	-7.09	23.09
	3	7.000	5.428	.665	-8.09	22.09
3	1	1.000	5.428	1.000	-14.09	16.09
	2	-7.000	5.428	.665	-22.09	8.09

Example2, GOLFCRD:

Brand 1	Brand 2	Brand 3	Brand 4
251.2	263.2	269.7	251.6
245.1	262.9	263.2	248.6
248.0	265.0	277.5	249.4
251.1	254.5	267.4	242.0
260.5	264.3	270.5	246.5
250.0	257.0	265.5	251.3
253.9	262.8	270.7	261.8
244.6	264.4	272.9	249.0
254.6	260.6	275.6	247.1
248.8	255.9	266.5	245.9

Recall sec.10.2, the conclusion of hypothesis test for comparing treatment means is to reject H0, so there is sufficient evidence that **at least two** of the four brands are different. ----**we can do multiple comparisons of means to explore which pairs brands are sig. different.**

1. Use Bonferroni's multiple comparisons procedure to rank treatment means with an overall confidence level of 95% to find out which pairs of brands are sig. different, and which pairs are not.

Question: 1. Which brand(s) is with the highest mean distance?
Which brand(s) is with the smallest mean distance?

2. How many pair-wise comparisons of means (μ_i, μ_j) are there?

3. List those pairs of means which are sig. different.

4. List those pairs of means which are not sig. different.

2. Use 95% confidence interval to estimate the mean distance traveled for balls manufactured by the brand with the highest rank.

Descriptives DISTANCE								
	N	**Mean**	**Std. Deviation**	**Std. Error**	**95% Confidence Interval for Mean**		**Minimum**	**Maximum**
					Lower Bound	**Upper Bound**		
1	10	250.780	4.7352	1.4974	247.393	254.167	244.6	260.5
2	10	261.060	3.8661	1.2226	258.294	263.826	254.5	265.0
3	10	269.950	4.5009	1.4233	266.730	273.170	263.2	277.5
4	10	249.320	5.2032	1.6454	245.598	253.042	242.0	261.8
Total	40	257.778	9.5497	1.5099	254.723	260.832	242.0	277.5

Multiple comparison SPSS output for GOLFCRD using Bonferroni's Method:

Post Hoc Tests

Multiple Comparisons Dependent Variable: DISTANCE Bonferroni						
(I) brand1	**(J) brand1**	**Mean Difference (I-J)**	**Std. Error**	**Sig.**	**95% Confidence Interval**	
					Lower Bound	**Upper Bound**
1	**2**	-10.2800(*)	2.0579	.000	-16.026	-4.534
	3	-19.1700(*)	2.0579	.000	-24.916	-13.424
	4	1.4600	2.0579	1.000	-4.286	7.206
2	**1**	10.2800(*)	2.0579	.000	4.534	16.026
	3	-8.8900(*)	2.0579	.001	-14.636	-3.144
	4	11.7400(*)	2.0579	.000	5.994	17.486
3	**1**	19.1700(*)	2.0579	.000	13.424	24.916
	2	8.8900(*)	2.0579	.001	3.144	14.636
	4	20.6300(*)	2.0579	.000	14.884	26.376
4	**1**	-1.4600	2.0579	1.000	-7.206	4.286
	2	-11.7400(*)	2.0579	.000	-17.486	-5.994
	3	-20.6300(*)	2.0579	.000	-26.376	-14.884
* The mean difference is significant at the .05 level.						

Example 3, SCOPOLAMINE,

Group 1 (Scopolamine)	5, 8, 8, 6, 6, 6, 6, 8, 6, 4, 5, 6
Group 2 (Placebo)	8, 10, 12, 10, 9, 7, 9, 10
Group 3 (No drug)	8, 9, 11, 12, 11, 10, 12, 12

Recall sec. 10.2, the conclusion of hypothesis test for comparing treatment means is to reject H0, so there is sufficient evidence that **at least two** of the three groups are different. ----**we can do multiple comparisons of means to explore which pairs groups are sig. different.**

Q: Use Bonferroni's multiple comparisons procedure to rank treatment means with an overall confidence level of 95% to find out which pairs of groups are sig. different, and which pairs are not.

Question: 1. Does Scopolamine have effect on people's memory?

2. How many pair-wise comparisons of means (μ_i, μ_j) are there?

3. List those pairs of means which are sig. different.

4. List those pairs of means which are not sig. different.

Post Hoc Tests

Multiple Comparisons
Dependent Variable: RECALL
Bonferroni

(I) GROUP	(J) GROUP	Mean Difference (I-J)	Std. Error	Sig.	95% Confidence Interval	
					Lower Bound	Upper Bound
1	2	-3.208(*)	.642	.000	-4.85	-1.56
	3	-4.458(*)	.642	.000	-6.10	-2.81
2	1	3.208(*)	.642	.000	1.56	4.85
	3	-1.250	.703	.263	-3.05	.55
3	1	4.458(*)	.642	.000	2.81	6.10
	2	1.250	.703	.263	-.55	3.05

* The mean difference is significant at the .05 level.

10.4 The randomized block design

- **The Randomized block design:**

1. There are ____matched sets of experimental units, called________**.**
   Each block consists of _____experimental units (where k is the number of treatments). Experimental units in each block should be as _____________as possible.
2. One experimental unit from each block is randomly assigned to________________,
3. Total responses n = _______.

Data layout: (two-way table)

	Treatment(k)			
Block(b)	1	2	……..	k
1				
2				
….				
b				

Example1: Drug effect. (when a drug effect is short-lived， no carryover effect; and the drug effect varies greatly from person to person) To study the effect of 3 different drugs, 6 people were used in the experiment. Each person took these three drugs at different time interval, then the reaction time were recorded.

Interest: the effect of ___________(treatment)
Data layout: (two-way table) k = ___, b =____, n = _____.

subject	Drug A	Drug B	Drug C
1	1.21	1.48	1.56
2	1.63	1.85	2.01
3	1.42	2.06	1.70
4	2.43	1.98	2.64
5	1.16	1.27	1.48
6	1.94	2.44	2.81

Example2: Diet Effect: Five litter piglets were used to investigate the effect of three diets on the weight gain for piglets. Three piglets were selected from each litter, hopefully the piglets from the same litter should be similar, then three different diets were randomly assigned to the three piglets in each litter. After one month, the average daily weight gain in pounds for each piglet was recorded.

Interest: the effect of __________(treatment)
Data layout: two-way table: k =_____, b = ______, n = _______

Litter	Diet A	Diet B	Diet C
Litter 1	1.47	1.53	1.39
Litter 2	1.33	1.67	1.50
Litter 3	1.48	1.63	1.37
Litter 4	1.46	1.76	1.48
Litter 5	1.41	1.68	1.48

Advantage of RBD: better to control variability by reducing the measure of error _______since the sampling variability of the E.U.s in each block will be reduced.

Note: ____

____

*Sum of Squares:

CM = Correction for mean =

Total sum of squares: SS(Total) = $\sum x_i^2 - CM$

Sum of squares for treatments (SST): ____________________________

Sum of squares for blocks (SSB): ________________________________

Sum of squares for error (SSE): __________________________________

(T_i : sum of observations for ith treatment B_i : sum of observations for ith block)

- ANOVA table for randomized block design (RBD):

Source	df	SS	MS	F

- Conditions required for a valid ANOVA F-test in RBD:

1. The b block are randomly selected and all k treatments are randomly assigned to k E.U.s in each block;
2. The distributions of observations corresponding to all n=b*k block-treatment combinations are approx.____________;
3. The b*k block-treatment distributions have __________variances.

If the assumptions are not satisfied, use nonparametric method.

- ANOVA analysis procedure for randomized block design (RBD):

1. Assumptions,

2. Summarize ANOVA table:

3. Conduct test of hypothesis to compare treatment means.

$H_0 : \mu_1 = \mu_2 = ... = \mu_k$ (All the ______________________are equal)

$H_a : \mu_i \neq \mu_j$ for some $i \neq j$ (At least two _______________ differ)

Test statistic:__________________,

Rejection region:________________________.

4. If possible, conduct test of hypothesis to compare block means.

$H_0 : \mu_1 = \mu_2 = ... = \mu_b$ (All the __________________are equal)

$H_a : \mu_i \neq \mu_j$ for some $i \neq j$ (At least two ____________ differ)

Test statistic:___________________,

Rejection region:______________________.

(Rejection of this null hypothesis gives statistical support to the utilization of the randomized block design.)

Examples of Randomized Block design:

Example1, OILRIGS,

A randomized block design was used to compare the average monthly number of rotary oil rigs running in the three states-California, Utah, and Alaska. Three months were randomly selected as blocks and the number of oil rigs running in each state in each month was recorded.
Conduct a test to determine if there is sufficient evidence to conclude that the mean number of oil rigs running differs among these three states. Use $\alpha = 0.05$.

Data:

Month/year	California	Utah	Alaska
Nov. 2000	27	17	11
Oct. 2001	34	20	14
Nov. 2001	36	15	14

E. U.: _______________
Treatment:_______________, (k =___)
Block:_________________ (b = ___)
Response: _______________________(n = b*k = ___)

SPSS output for Example1, OILRIGS,

Estimated Marginal Means

1. Grand Mean

Dependent Variable: numrigs

Mean	Std. Error	95% Confidence Interval	
		Lower Bound	Upper Bound
20.889	.949	18.253	23.525

2. monthyr

Dependent Variable: numrigs

monthyr	Mean	Std. Error	95% Confidence Interval	
			Lower Bound	Upper Bound
Nov2000	18.333	1.644	13.768	22.899
Nov2001	21.667	1.644	17.101	26.232
Oct2001	22.667	1.644	18.101	27.232

3. state

Dependent Variable: numrigs

state	Mean	Std. Error	95% Confidence Interval	
			Lower Bound	Upper Bound
AL	13.000	1.644	8.435	17.565
CAL	32.333	1.644	27.768	36.899
UT	17.333	1.644	12.768	21.899

Tests of Between-Subjects Effects

Dependent Variable: numrigs

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	648.444(a)	4	162.111	19.986	.007
Intercept	3927.111	1	3927.111	484.164	.000
monthyr	30.889	2	15.444	1.904	.262
state	617.556	2	308.778	38.068	.002
Error	32.444	4	8.111		
Total	4608.000	9			
Corrected Total	680.889	8			

a R Squared = .952 (Adjusted R Squared = .905)

Multiple Comparisons

Dependent Variable: numrigs

Bonferroni

(I) state	(J) state	Mean Difference (I-J)	Std. Error	Sig.	95% Confidence Interval	
					Lower Bound	Upper Bound
AL	CAL	-19.33*	2.325	.003	-28.54	-10.12
	UT	-4.33	2.325	.408	-13.54	4.88
CAL	AL	19.33*	2.325	.003	10.12	28.54
	UT	15.00*	2.325	.009	5.79	24.21
UT	AL	4.33	2.325	.408	-4.88	13.54
	CAL	-15.00*	2.325	.009	-24.21	-5.79

Based on observed means.

*. The mean difference is significant at the .05 level.

Example2, GOLFRBD,

To compare the mean distances associated with four different brands of golf balls. Use a random sample of 10 golfers with each golfer using a driver to hit four brand balls in a random sequence. Conduct a test of the research hypothesis that the brand mean distances differ. Use α =0.05.

Data:(two-way table)

Golfer(Block)	Brand A	Brand B	Brand C	Brand D
1	202.4	203.2	223.7	203.6
2	242.0	248.7	259.8	240.7
3	220.4	227.3	240.0	207.4
4	230.0	243.1	247.7	226.9
5	191.6	211.4	218.7	200.1
6	247.7	253.0	268.1	244.0
7	214.8	214.8	233.9	195.8
8	245.4	243.6	257.8	227.9
9	224.0	231.5	238.2	215.7
10	252.2	255.2	265.4	245.2

E. U.:______________;

Blocks:______________ (b =___)

Treatments:_______________, (k = __);

Response variable: _______________(n = b*k =____)

ANOVA table:

Source df SS MS F

SPSS output for Example2: GOLFRBD,

Tests of Between-Subjects Effects

Dependent Variable: DISTANCE

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	15372.539(a)	12	1281.045	63.276	.000
Intercept	2145032.910	1	2145032.910	105952.598	.000
GOLFER	12073.882	9	1341.542	66.265	.000
BRAND	3298.657	3	1099.552	54.312	.000
Error	546.621	27	20.245		
Total	2160952.070	40			
Corrected Total	15919.160	39			

a R Squared = .966 (Adjusted R Squared = .950)

Estimated Marginal Means

BRAND Dependent Variable: DISTANCE				
BRAND	**Mean**	**Std. Error**	**95% Confidence Interval**	
			Lower Bound	**Upper Bound**
A	227.050	1.423	224.131	229.969
B	233.180	1.423	230.261	236.099
C	245.330	1.423	242.411	248.249
D	220.730	1.423	217.811	223.649

Post Hoc Tests

BRAND

Multiple Comparisons Dependent Variable: DISTANCE Bonferroni						
(I) BRAND	**(J) BRAND**	**Mean Difference (I-J)**	**Std. Error**	**Sig.**	**95% Confidence Interval**	
					Lower Bound	**Upper Bound**
A	**B**	-6.1300(*)	2.01222	.031	-11.8586	-.4014
	C	-18.2800(*)	2.01222	.000	-24.0086	-12.5514
	D	6.3200(*)	2.01222	.024	.5914	12.0486
B	**A**	6.1300(*)	2.01222	.031	.4014	11.8586
	C	-12.1500(*)	2.01222	.000	-17.8786	-6.4214
	D	12.4500(*)	2.01222	.000	6.7214	18.1786
C	**A**	18.2800(*)	2.01222	.000	12.5514	24.0086
	B	12.1500(*)	2.01222	.000	6.4214	17.8786
	D	24.6000(*)	2.01222	.000	18.8714	30.3286
D	**A**	-6.3200(*)	2.01222	.024	-12.0486	-.5914
	B	-12.4500(*)	2.01222	.000	-18.1786	-6.7214
	C	-24.6000(*)	2.01222	.000	-30.3286	-18.8714
Based on observed means.						

Exercise: Below is an incomplete ANOVA table for RBD.

source	df	SS	MS	F
Drug(treatment)	2	329		
Patient(block)	9	1207		
Error				
Total	29	1591		

1. Complete ANOVA table.
2. How many treatments are involved in this experiment?
3. How many blocks are involved in this experiment?
4. How much is the MSE?
5. How much is the F test statistic used to compare drug means?
6. How much is the F test statistic used to compare patient means?
7. Write down the rejection region for hypothesis test to compare drug means.
8. Write down the rejection region for hypothesis test to compare patient means.

10.5 factorial Experiments

For single- factor experiment: The treatments are ______________of this single factor.
For more than one factor experiment: The treatments are the complete____________________.
(**A ____)**

Here we just talk about **two factor** experiment. (A and B, factor A has a levels, factor B has b levels) we call this $a \times b$ **factorial experiment**. (the # of treatments is__________**)**

Example1, A experiment is interested in studying the effect of display types (normal, normal plus, normal twice) and price levels (regular, reduced, cost) on unit sales for supermarkets.
____________**factorial experiment**. (the # of treatments is _______)

Example2, A experiment is designed to investigate the effect of the gender (male, female) of firefighters and the weight (light, heavy) on the length of time required for a firefighter to perform a particular firefighting task.
_____________**factorial experiment**. (the # of treatments is _____)

If we utilize a CRD to conduct a factorial experiment with a*b treatments, we can use ANOVA F-test to compare a*b combination treatment means. If the H_0 is rejected, we conclude some differences exist among the treatment means, it indicates that factor A and B somehow have effect on mean response, then we need to answer the following questions.

1. Do ____________ affect the response, or ____________?
2. If both, do they affect the response ______________ or do they _________________ to affect the response?

In order to determine the nature of the treatment effect, we need to break the **treatment variability** into **three** parts:_______________________________, _______________________________, and ____.

SST = _______________________________
(ab- 1) = _________________________________

SS(Total) = ___________________________
(n - 1) = _______________________________

- **ANOVA Table for a $a \times b$ factorial experiment:**

Source	df	SS	MS	F

- **Conditions required for a valid ANOVA F-test for** $a \times b$ Factorial Experiment**:**

1. ____________________-samples of experimental units are associated with each treatment;
2. The response distribution for each factor-level combination (treatment) is____________;
3. The response variance is ________________for all treatments.

- Procedure of ANOVA analysis for $a \times b$ Factorial Experiment (Ordered F-test):

Step 1, Test ________________________means

(Question: Do factor A and factor B have effect on mean response?)

________________________________ (____)

$H_a : \mu_i \neq \mu_j$ for some $i \neq j$ **(____)**

Test statistic:______________________________,

Rejection region:____________________________.

If the conclusion leads to______________, there is sufficient evidence that Factor A and B _______ effect on mean response. **Go to step 2** **to test if they interact significantly to affect mean response.**

If the conclusion leads to________________, STOP here and conclude____________________ ___________________________________ .

Step 2, Test ________________between A and B

(Question: Do factor A and factor B interact significantly to affect mean response?)

H_0: Factor A and B __________________to affect the mean response;

H_a: Factor A and B ________________to affect the mean response;

Test statistic:________________________,

Rejection region:_________________________.

If the conclusion leads to_____________, we conclude that________________________________ _____________________________________.

If the conclusion leads to____________________, it implies there is no interaction between factors A and B. **Go to step 3.**

Step 3, Test _______________for both factors A and B

1: Test main effect of factor A
(Question: Does factor A have sig. effect on mean response?)

___________________________ (____________________________________)

$H_a : \mu_i \neq \mu_j$ for some $i \neq j$ **(At least two of the a treatment means differ)**

Test statistic:______________________,

Rejection region:____________________.

Conclusion.

2: Test main effect of factor B
(Question: Does factor B have sig. effect on mean response?)

____________________________ (_____________________________________)

$H_a : \mu_i \neq \mu_j$ for some $i \neq j$ **(At least two of the b treatment means differ)**

Test statistic:_________________________,

Rejection region:_________________________________.

Conclusion.

Based on ordered F-test, give an overall conclusion to tell how factor A and B affect response.

$a \times b$ Factorial Experiments Examples:

Example1: GOLFFAC1,
Suppose the USGA tests four different brands (A, B, C, D) of golf balls and two different clubs (driver, 5-iron) in a 2×4 factorial design. Each of the eight Brand-Club combinations (treatments) is randomly and independently assigned to four experimental units, each experimental unit consisting of a specific position in the sequence of hits by Iron Byron. The distance response is recorded for each of the 32 hits.

Data: a = , b = , n =

	Driver	Five-iron
Brand A	226.4, 232.6, 234.0, 220.7	163.8, 179.4, 168.6, 173.4
Brand B	238.3, 231.7, 227.7, 237.2	184.4, 180.6, 179.5, 186.2
Brand C	240.5, 246.9, 240.3, 244.7	179.0, 168.0, 165.2, 156.5
Brand D	219.8, 228.7, 232.9, 237.6	157.8, 161.8, 162.1, 160.3

Factors:
Treatments:
Response variable:

a. Conduct an analysis of variance on the data. Summarize the results in an AVOVA table.

b. Conduct the appropriate F-test at $\alpha = 0.10$ **to determine how club and brand affect mean distance golf balls travel.**

SPSS output for GOLFFAC1,

Between-Subjects Factors

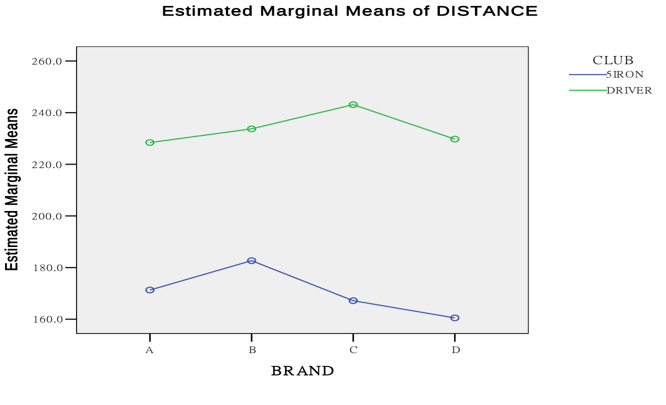
		N
CLUB	**5IRON**	16
	DRIVER	16
BRAND	**A**	8
	B	8
	C	8
	D	8

Tests of Between-Subjects Effects

Dependent Variable: DISTANCE

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	33659.809(a)	7	4808.544	140.354	.000
Intercept	1306778.611	1	1306778.611	38142.983	.000
CLUB	32093.111	1	32093.111	936.752	.000
BRAND	800.736	3	266.912	7.791	.001
CLUB * BRAND	765.961	3	255.320	7.452	.001
Error	822.240	24	34.260		
Total	1341260.660	32			
Corrected Total	34482.049	31			

a R Squared = .976 (Adjusted R Squared = .969)



Example 2, GOLFFAC2,

Suppose the USGA tests four different brands (E, F, G, H) of golf balls and two different clubs (driver, 5-iron) in a 2×4 factorial design. Each of the eight Brand-Club combinations (treatments) is randomly and independently assigned to four experimental units, each experimental unit consisting of a specific position in the sequence of hits by Iron Byron. The distance response is recorded for each of the 32 hits.

Data: a = 2, b = 4, n = 32

	Driver	Five-iron
Brand E	238.6, 241.9, 236.6, 244.9	165.2, 156.9, 172.2, 163.2
Brand F	261.4, 261.3, 254.0, 259.9	179.2, 171.0, 178.0, 182.7
Brand G	264.7, 262.9, 253.5, 255.6	189.0, 191.2, 191.3, 180.5
Brand H	235.4, 239.8, 236.2, 237.5	171.4, 159.3, 156.6, 157.4

Factors:

Treatments:

Response variable:

a. conduct an analysis of variance on the data. Summarize the results in an AVOVA table.

b. Conduct the appropriate ANOVA **ordered** F-test at $\alpha = 0.10$ to find out how club and brand affect mean distance.

SPSS output for Example 2, GOLFFAC2,

Warnings
Post hoc tests are not performed for CLUB because there are fewer than three groups.

Between-Subjects Factors

		N
CLUB	**5IRON**	16
	DRIVER	16
BRAND	**E**	8
	F	8
	G	8
	H	8

Tests of Between-Subjects Effects

Dependent Variable: DISTANCE

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	49959.375(a)	7	7137.054	290.120	.000
Intercept	1423532.828	1	1423532.828	57866.453	.000
CLUB	46443.900	1	46443.900	1887.939	.000
BRAND	3410.316	3	1136.772	46.210	.000
CLUB * BRAND	105.158	3	35.053	1.425	.260
Error	590.408	24	24.600		
Total	1474082.610	32			
Corrected Total	50549.782	31			

a R Squared = .988 (Adjusted R Squared = .985)

Estimated Marginal Means

1. Grand Mean

Dependent Variable: DISTANCE

Mean	Std. Error	95% Confidence Interval	
		Lower Bound	Upper Bound
210.916	.877	209.106	212.725

2. CLUB

Dependent Variable: DISTANCE

CLUB	Mean	Std. Error	95% Confidence Interval	
			Lower Bound	Upper Bound
5IRON	172.819	1.240	170.260	175.378
DRIVER	249.013	1.240	246.453	251.572

3. BRAND

Dependent Variable: DISTANCE

BRAND	Mean	Std. Error	95% Confidence Interval	
			Lower Bound	Upper Bound
E	202.438	1.754	198.818	206.057
F	218.438	1.754	214.818	222.057
G	223.588	1.754	219.968	227.207
H	199.200	1.754	195.581	202.819

4. CLUB * BRAND

Dependent Variable: DISTANCE

CLUB	BRAND	Mean	Std. Error	95% Confidence Interval	
				Lower Bound	Upper Bound
5IRON	E	164.375	2.480	159.257	169.493
	F	177.725	2.480	172.607	182.843
	G	188.000	2.480	182.882	193.118
	H	161.175	2.480	156.057	166.293
DRIVER	E	240.500	2.480	235.382	245.618
	F	259.150	2.480	254.032	264.268
	G	259.175	2.480	254.057	264.293
	H	237.225	2.480	232.107	242.343

Post Hoc Tests

BRAND

Multiple Comparisons
Dependent Variable: DISTANCE
Bonferroni

(I) BRAND	(J) BRAND	Mean Difference (I-J)	Std. Error	Sig.	95% Confidence Interval	
					Lower Bound	Upper Bound
E	F	-16.0000(*)	2.47994	.000	-23.1300	-8.8700
	G	-21.1500(*)	2.47994	.000	-28.2800	-14.0200
	H	3.2375	2.47994	1.000	-3.8925	10.3675
F	E	16.0000(*)	2.47994	.000	8.8700	23.1300
	G	-5.1500	2.47994	.292	-12.2800	1.9800
	H	19.2375(*)	2.47994	.000	12.1075	26.3675
G	E	21.1500(*)	2.47994	.000	14.0200	28.2800
	F	5.1500	2.47994	.292	-1.9800	12.2800
	H	24.3875(*)	2.47994	.000	17.2575	31.5175
H	E	-3.2375	2.47994	1.000	-10.3675	3.8925
	F	-19.2375(*)	2.47994	.000	-26.3675	-12.1075
	G	-24.3875(*)	2.47994	.000	-31.5175	-17.2575

Based on observed means.

* The mean difference is significant at the .05 level.