Chapter 9: Inferences based on Two samples:

Confidence intervals and tests of hypotheses

9.1 The target parameter

_____________: difference between two population means

_____________: difference between two population proportions

_____________: ratio of two population variances

9.2 Comparing two population means: independent sampling

Case 1, Large samples

- **Conditions required for valid large sample:**

1. two sample ___________and ______________selected from two independent population,

2.____________ and $n_2 \geq 30$

- **Sampling distribution of ($\bar{x}_1 - \bar{x}_2$) is approximately normal with:**

Mean: $\mu_{(\bar{x}_1 - \bar{x}_2)} =$ ________________

Standard error: $\sigma_{(\bar{x}_1 - \bar{x}_2)} = \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} \approx$ ______________

- **(1- α) 100% Confidence interval for $(\mu_1 - \mu_2)$ (the difference of two population means):**

____

Interpret: We are (1- α) 100% confident that the true ________________________ between these two populations will falls in this interval.

Example1, DIETSTUDY,

To investigate the effect of a new low-fat diet on weight loss, two random samples of 100 people each are selected. One group of 100 is placed on the low-fat diet, while the other group with regular diet. For each person, the amount of weight lost (or gained) in 3-week period is recorded.

Diet	Weight loss
Low-fat diet (1)	8, 21, 13, …………….., 10 (100 observations)
Regular diet (2)	6, 14, 4, ………………, 8 (100 observations)

Q: Form a 95% confidence interval for ($\mu_1 - \mu_2$), the difference between the population mean weight losses for the two diets. Interpret the result.

Group Statistics					
	DIET	**N**	**Mean**	**Std. Deviation**	**Std. Error Mean**
WTLOSS	**LOWFAT**	100	9.31	4.668	.467
	REGULAR	100	7.40	4.035	.404

- **How can we make inference based on (1- α) 100% Confidence interval for $(\mu_1 - \mu_2)$?**

If the confidence interval________________, it implies that there is _____________**difference** between these two population means;
If the confidence interval________________, it implies that there is ____________**difference** between these two population means.

Example: A confidence interval for $(\mu_1 - \mu_2)$ is (-10, 4), what inference can we make?

A confidence interval for $(\mu_1 - \mu_2)$ is (-18, -9), what inference can we make?

A confidence interval for $(\mu_1 - \mu_2)$ is (3.6, 12.4), what inference can we make?

Hypothesis test for ($\mu_1 - \mu_2$):

- **Critical-value approach:**

1.

____

2. **significance level** α

3. **test statistic:**

4. **rejection region :** ____________________when $H_a: \mu_1 - \mu_2 \neq D_0$

_______________________ when $H_a: \mu_1 - \mu_2 < D_0$

_______________________ when $H_a: \mu_1 - \mu_2 > D_0$

5. **conclusion:** if the value of test statistic **falls in R.R, reject** H_0, and conclude that at α level, there is **sufficient** evidence to **conclude** H_a **is true**.

if the value of test statistic **does not fall in R.R, do not reject** H_0, and conclude that at α level, there is **insufficient** evidence to **conclude** H_a **is true.**

- **P-value approach:**

1. $H_0: \mu_1 - \mu_2 = D_0$
$H_a: \mu_1 - \mu_2 \neq D_0 \quad (or \quad \mu_1 - \mu_2 < D_0 \quad or \quad \mu_1 - \mu_2 > D_0)$

2. **significance level** α

3. **test statistic** $z_0 = \dfrac{(\bar{x}_1 - \bar{x}_2) - D_0}{\sqrt{\dfrac{S_1^2}{n_1} + \dfrac{S_2^2}{n_2}}}$

4. **p-value** = ________________ when $H_a: \mu_1 - \mu_2 \neq D_0$;

__________________ when $H_a: \mu_1 - \mu_2 < D_0$;

___________________ when $H_a: \mu_1 - \mu_2 > D_0$.

5. **Conclusion**: if **p-value is** _________________α , reject H_0, and conclude that at α level, there is sufficient evidence to conclude H_a is true.

If p-value is no less than α , _________________ H_0, and conclude that at α level, there is insufficient evidence to conclude H_a is true.

Examples for comparing two population means: independent, large-samples:

Example1, DIETSTUDY,

To investigate the effect of a new low-fat diet on weight loss, two random samples of 100 people each are selected. One group of 100 is placed on the low-fat diet, while the other group with regular diet. For each person, the amount of weight lost (or gained) in 3-week period is recorded.

Diet	Weight loss
Low-fat diet (1)	8, 21, 13, …………….., 10 (100 observations)
Regular diet (2)	6, 14, 4, ………………, 8 (100 observations)

a. At α=0.05, conduct a test of hypothesis to determine whether the mean weight loss for low-fat diet is **different from** that of regular diet.

b. At α=0.05, conduct a test of hypothesis to determine whether the mean weight loss for low-fat diet is **greater than** that of regular diet.

SPSS output for DIETSTUDY,

Group Statistics

	DIET	N	Mean	Std. Deviation	Std. Error Mean
WTLOSS	LOWFAT	100	9.31	4.668	.467
	REGULAR	100	7.40	4.035	.404

Independent Samples Test

		Levene's Test for Equality of Variances		t-test for Equality of Means						
		F	Sig.	t	df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	95% Confidence Interval of the Difference	
									Lower	Upper
WTLOSS	Equal variances assumed	1.367	.244	3.095	198	.002	1.910	.617	.693	3.127
	Equal variances not assumed			3.095	193.940	.002	1.910	.617	.693	3.127

Case 2, Small samples with equal variances

- **Conditions required for valid small sample:**

1. The two samples are ____________and_____________selected from the two target population,
2. Both sampled populations have distributions that are approx.____________,
3. The population variances are____________. ($\sigma_1^2 = \sigma_2^2$)
4. Sample size is small (___________________).

Since these two populations have equal variance, ($\sigma_1^2 = \sigma_2^2$), it is reasonable to construct a _________________________________for use in confidence intervals and test statistics.

____

- (1-α)100% confidence interval for ($\mu_1 - \mu_2$):

_______________________________ $t_{\alpha/2}$ with $df = n_1 + n_2 - 2$.

- Hypothesis test for ($\mu_1 - \mu_2$):

1. $H_0 : \mu_1 - \mu_2 = D_0$
   $H_a : \mu_1 - \mu_2 \neq D_0$ $(or \quad \mu_1 - \mu_2 < D_0 \quad or \quad \mu_1 - \mu_2 > D_0)$

2. level of significance α ;

3. test statistic: $t =$ with $df =$ ______________

4. rejection region : ___________________ when $H_a : \mu_1 - \mu_2 \neq D_0$

   ___________________ when $H_a : \mu_1 - \mu_2 < D_0$

   ___________________ when $H_a : \mu_1 - \mu_2 > D_0$;

5. conclusion: if the value of test statistic falls in R.R, _________ H_0 , and conclude that at α level, there is **sufficient** evidence to conclude H_a is true.

   if the value of test statistic does not fall in R.R, ____________ H_0 , and conclude that at α level, there is **insufficient** evidence to conclude H_a is true.

Examples for comparing two population means: independent, small-samples:

Example1: READING,

Suppose we wish to compare a new method of teaching reading to "slow learners" to the current standard method. The response variable is the reading test score after 6 months. 22 slow learners are randomly selected, 10 are taught by the new method, 12 by the standard method. The test score is listed below.

New method (1)	80, 80, 79, 81, 76, 66, 71, 76, 70, 85
Standard method (2)	79, 62, 70, 68, 73, 76, 86, 73, 72, 68, 75, 66

a. Use a 95% confidence interval to estimate the true mean difference between the test score for the new method and the standard method. Interpret the interval.

b. Conduct a test of hypothesis to determine whether the standard method leads to a lower test score than new method. Use $\alpha = 0.05$.

SPSS output for READING,

Group Statistics

	METHOD	N	Mean	Std. Deviation	Std. Error Mean
SCORE	NEW	10	76.40	5.835	1.845
	STD	12	72.33	6.344	1.831

Independent Samples Test

		Levene's Test for Equality of Variances		t-test for Equality of Means						
		F	Sig.	t	df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	95% Confidence Interval of the Difference	
									Lower	Upper
SCORE	Equal variances assumed	.002	.967	1.552	20	.136	4.067	2.620	-1.399	9.533
	Equal variances not assumed			1.564	19.769	.134	4.067	2.600	-1.360	9.493

Case 3, Small samples with unequal variance:

Conditions:

1. two samples are randomly and independently selected from the two target population,
2. both sampled populations are approx. normal,
3. the populations variance are not equal ($\sigma_1^2 \neq \sigma_2^2$).

Procedure is on textbook P422-423.

9.3 Comparing two population means: paired difference experiments

Two sampling comparing:

Example1: To investigate the effect of new teaching method on reading.

1. Randomly select 22 slow learner students, 10 are assigned to new method, while the other 12 are assigned to the standard method, the response variable is the reading test score after 6 months. (independent sampling)
2. 8 pairs slow learner are selected, not randomly, two learners in each pair with the similar reading IQs; in each pair, one use new method, the other one use standard method, then the paired test score difference could be used to make inference about $(\mu_1 - \mu_2)$.

Example2: To investigate the effect of new protein diet on weight loss.

1. Two random samples of 100 people each are selected. One group of 100 is placed on the low-fat diet, while the other group with regular diet. For each person, the amount of weight lost (or gained) in 3-week period is recorded. (independent sampling)
2. FDA randomly choose five individuals with regular diet and record their weight (in pounds), then instruct them to follow the protein diet for three weeks. At the end of this period, their weights are recorded again. The paired weight differences between these two diets could be used to make inference about $(\mu_1 - \mu_2)$.

(two subjects in each pair with similar level, then assign treatments, to see the effect)

- Paired difference experiment: Each pair has two ______________experiment al units, _______________are paired and the _________________are analyzed.

- __________: making comparisons within groups of similar experimental units.
- Paired difference experiment is a simple example of randomized block design.

(Read textbook P432, 433)

Example data (NEW PROTEIN DIET):

Person	**Weight before (1)**	**Weight after (2)**		
1	**148**	**141**		
2	**193**	**188**		
3	**186**	**183**		
4	**195**	**189**		
5	**202**	**198**		

Note: The variable we are interested is __________________ .

- Inference based on paired difference (large sample):

Conditions required for a valid large-sample inference about μ_D:

1. A random sample of ____________is selected from the target population of differences;

2. The sample size is n_D_________.

Paired difference (1-α)100% confidence interval for $\mu_D = (\mu_1 - \mu_2)$:

Paired difference Test of hypothesis for $\mu_D = (\mu_1 - \mu_2)$:

1.

2. significance level α;

3. test statistic: $z = \dfrac{\overline{x}_D - D_0}{\sigma_D / \sqrt{n_D}} \approx$

4. rejection region: $|Z| > Z_{\alpha/2}$ when H_a: ____________

$Z < -Z_\alpha$ when H_a: ____________

$Z > Z_\alpha$ when H_a: ____________

5. conclusion.

Examples of making inference based on paired difference (large sample):

Example, To investigate which supermarket (A or B) has the lower prices in town, a agency randomly selected 100 items **common** to each of the two supermarkets and recorded the prices charged by each supermarket. The summary results are provided below.

$\overline{x}_A = 2.09 \quad \overline{x}_B = 1.99 \quad \overline{x}_D = 0.10$

$S_A = 0.24 \quad S_B = 0.19 \quad S_D = 0.03$

a. Form a 95% confidence interval for $\mu_D = \mu_A - \mu_B$. Interpret the result.

b. Conduct a test of hypothesis to determine whether the mean price for supermarket B is **cheaper than** that for supermarket B? Use $\alpha = 0.05$.

- Inference based on paired difference (<u>small sample</u>):

- **Conditions required for a valid small-sample inference about μ_D:**

1. A ________________of difference is selected from the target population of differences;
2. The population of differences is approximately _______________distributed;
3. The sample size $n_D < 30$.

1. (1-α)100% confidence interval for Paired difference $\mu_D = (\mu_1 - \mu_2)$:

__________________________________ , $t_{\alpha/2}$ with $df =$___________

2. Test of hypothesis for Paired difference $\mu_D = (\mu_1 - \mu_2)$:

1. $H_0 : \mu_D = D_0$
   $H_a : \mu_D \neq D_0 \quad (or \quad H_a : \mu_D < D_0 \quad or \quad H_a : \mu_D > D_0)$
2. significance level α;
3. test statistic: $t =$
4. rejection region: _________________when $H_a : \mu_D \neq D_0$

   _________________ when $H_a : \mu_D < D_0$

   __________________ when $H_a : \mu_D > D_0$
5. conclusion.

Examples of making inference based on paired difference (small sample):

Example 1, NEW PROTEIN DIET: To investigate a new protein diet on weight-loss, FDA randomly choose five individuals and record their weight (in pounds), then instruct them to follow the protein diet for three weeks. At the end of this period, their weights are recorded again.

person	Weight before (1)	Weight after (2)	
1	148	141	
2	193	188	
3	186	183	
4	195	189	
5	202	198	

a. Calculate a 95% confidence interval for the difference between the mean weights before and after the diet is used. Interpret the interval.

b. Do the data provide sufficient evidence that the protein diet has effect on the weight loss? Use $\alpha = 0.05$. (p-value = ?)

SPSS output for Example1, NEW PROTEIN DIET,

Paired Samples Statistics

		Mean	N	Std. Deviation	Std. Error Mean
Pair 1	W1	184.80	5	21.347	9.547
	W2	179.80	5	22.354	9.997

Paired Samples Test

		Paired Differences					t	df	Sig. (2-tailed)
		Mean	Std. Deviation	Std. Error Mean	95% Confidence Interval of the Difference				
					Lower	Upper			
Pair 1	W1 - W2	5.000	1.581	.707	3.037	6.963	7.071	4	.002

SPSS output for Example2, PAIREDSCORES,

Paired Samples Statistics

		Mean	N	Std. Deviation	Std. Error Mean
Pair 1	NEW	76.00	8	6.928	2.449
	STD	71.63	8	7.009	2.478

Paired Samples Test

		Paired Differences					t	df	Sig. (2-tailed)
		Mean	Std. Deviation	Std. Error Mean	95% Confidence Interval of the Difference				
					Lower	Upper			
Pair 1	NEW - STD	4.375	1.685	.596	2.966	5.784	7.344	7	.000

Example 2, PAIREDSCORES,

To investigate the effect of a new teaching method on improving reading test score, **8 pairs** slow learner are selected, not randomly, two learners in each pair with the similar reading IQs; in each pair, one use new method, the other one use standard method. Then after 6 months, the test scores are recorded.

Pair	New method (1)	Standard method (2)		
1	77	72		
2	74	68		
3	82	76		
4	73	68		
5	87	84		
6	69	68		
7	66	61		
8	80	76		

a. Construct a 95% confidence interval to estimate the difference of mean test scores between new method and standard method. Interpret the result

b. Do the data provide sufficient evidence that the new method leads to **higher** test scores than the standard method? Use $\alpha = 0.05$. (p-value = ?)

9.4 Comparing two population proportions: independent sampling

Conditions required for valid large-sample inferences about $(p_1 - p_2)$ **:**

1. The two samples are <u>randomly</u> and <u>independently</u> selected from the two target populations.
2. The sample size n_1 and n_2 are both large. (This condition will be satisfied if both____________________________________.)

Under large sample size, by the Central Limit Theorem,
the sampling distribution of $(\hat{p}_1 - p_2)$ **is approximately normal with:**

mean:

standard deviation:

- Large sample 100(1- α)% confidence interval for $(p_1 - p_2)$:

- Large –sample test of hypothesis about $(p_1 - p_2)$:

1.

2. Level of significance α ;

3. Test statistic:

4. rejection region : $Z < -Z_{\alpha/2}$ *or* $Z > Z_{\alpha/2}$ when $H_a : p_1 - p_2 \neq 0$

$Z < -Z_{\alpha}$ when $H_a : p_1 - p_2 < 0$

$Z > Z_{\alpha}$ when $H_a : p_1 - p_2 > 0$;

5. conclusion.

Examples for comparing two population proportions (large-sample):

Example: Smoking Survey,
Suppose the American cancer Society randomly samples 1500 adults in 1995 and then sampled 1750 adults in 2005 to do a smoking survey to determine whether there was evidence that the percentage of smokers had decreased.

1995 (1)	2005 (2)
$n_1 = 1500$	$n_2 = 1750$
$x_1 = 555$	$x_2 = 578$

Define: **____:** the true proportion of adult smokers in 1995;
_____: the true proportion of adult smokers in 2005.

a. Give a point estimate of the percentage difference of adult smokers between 1995 and 2005?

b. Do the data indicate that the proportion of adult smokers **decreased** over this 10-year period? Use $\alpha = 0.05$.

c. Form a 95% confidence interval for $(p_1 - p_2)$ to estimate the extent of the decrease. Interpret it.