The <u>binomial distribution</u> is a probability distribution that is used to model the probability that a certain number of "successes" occur during a certain number of trials.

Example 1: Number of Side Effects from Medications

Medical professionals use the binomial distribution to model the probability that a certain number of patients will experience side effects because of taking new medications.

For example, suppose it is known that 5% of adults who take a certain medication experience negative side effects. We can find the probability that more than a certain number of patients in a random sample of 100 will experience negative side effects.

This gives medical professionals an idea of how likely it is that more than a certain number of patients will experience negative side effects.

Example 2: Number of Fraudulent Transactions

Banks use the binomial distribution to model the probability that a certain number of credit card transactions are fraudulent.

For example, suppose it is known that 2% of all credit card transactions in a certain region are fraudulent. If there are 50 transactions per day in a certain region, we can find the probability that more than a certain number of fraudulent transactions occur in a given day:

This gives banks an idea of how likely it is that more than a certain number of fraudulent transactions will occur in a given day.

Example 3: Number of River Overflows

Park systems use the binomial distribution to model the probability that rivers overflow a certain number of times each year due to excessive rain.

For example, suppose it is known that a given river overflows during 5% of all storms. If there are 20 storms in a given year, we can find the probability that the river overflows a certain number of times:

This gives the parks departments an idea of how many times they may need to prepare for overflows throughout the year.

Example 4: Shopping Returns per Week

Retail stores use the binomial distribution to model the probability that they receive a certain number of shopping returns each week.

For example, suppose it is known that 10% of all orders get returned at a certain store each week. If there are 50 orders that week, we can find the probability that the store receives more than a certain number of returns that week:

This gives the store an idea of how many customer service reps they need to have in the store that week to handle returns.

Discrete random variable problems

1. How much money on average will an insurance company make off of a 1-year life insurance policy worth \$50,000, if they charge \$800.00 for the policy, and you have a 0.9999 probability of surviving the year?

What is your expected value on the following game? It involves a single roll of a die. If you roll a 1,
 or 5, you make \$1, \$3, or \$5 respectively. If you roll 2, 4, or 6, you lose \$2, \$4, or \$4 respectively.

3. A pawn shop owner buys an old Pentax camera from a customer. There is a 35% chance of making \$200 off the deal, a 45% chance of breaking even, or a 20% chance of losing \$50. What is the expected profit?

4. In the game roulette, there are 18 red numbers, 18 black numbers, and two green numbers. They
are all equally likely. If a person makes a \$5 bet on red and a red number comes out, he/she will make a
\$5 profit. If another color comes out, the person will lose his original \$5 bet. What is the expected value for this bet?

Binomial random variable problems

5. Is the following a binomial experiment? If not, state the condition that is violated: We plan to count the number of hits out of five first times at bat ('first time at bat' means the first time at bat during the course of a game) for a major league baseball player. On any given first time at bat there is a 28% chance the player gets a hit.

6. Is the following a binomial experiment? If not, state the condition that is violated: We plan to count the number of red cards out of a ten-card hand drawn from a well shuffled deck. Half the deck or 26 out of 52 cards are red in a standard deck of cards.

7. Is the following a binomial experiment? If not, state the condition that is violated: We plan to count the number of correct guesses on a quiz that has four multiple choice questions (with four answer choices each) and two true false questions.

8. Is the following a binomial experiment? If not, state the condition that is violated: We plan to ask immigrants if they think America has lived up to their expectations. We will ask as many people as it takes until we have 30 people who say yes—America has lived up to their expectations.

9. Is the following a binomial experiment? If not, state the condition that is violated: We plan to ask
500 people to roll a six sided die and record the number that turns up on each roll. Each number on the die has a 1/6th chance of turning up on the die.

10. If you flip a fair coin 30 times, what is the probability that you get exactly 15 heads?

- 11. What is the probability that when making random guesses on a fifty question multiple choice exam, with four answer choices for each question, that you miss only thirty-five of the questions?
- 12. A car dealership sells extended warranties on 50% of its vehicles. What is the probability that only 10 cars sales out of 30 sold purchase the extended warranty?
- 13. An IRS agent must review a stack of 9 randomly chosen tax returns for mistakes. If mistakes occur in 10% of the returns, what is the probability that there is at most one return with a mistake?

14. A marksman can hit the ten-spot on a target at 50 yards 89% of the time. If he makes 6 shots, find the probability that he lands 4 of them in the ten-spot.

15. A marksman can hit the ten-spot on a target at 50 yards 89% of the time. If he makes 6 shots, find the probability that he lands at least 5 of them in the ten-spot.

16. At FIU, 94% of the graduates, entering the workforce, find work within six months of their graduation. What is the probability that among 15 graduates exactly 12 of them find jobs within six months of graduation?

17. At FIU, 94% of the graduates, entering the workforce, find work within six months of their graduation. What is the probability that among 15 graduates more than two of them find jobs within six months of graduation?

18. If 25% of eleven-year-old children have no decayed, missing, or filled (DMF) teeth, find the probability that in a sample of 20 children there will be:

- a. Exactly 3 of them with no DMF teeth.
- b. Less than 3 with no DMF teeth.
- c. 3 or more with no DMF teeth.

19. Twenty-two percent of homes no longer use a traditional land line for telephone service. If 50 homes are randomly surveyed, what is the average number of homes we'd expect to find that do not have a land line? What is the standard deviation for the number of homes that do not have a land line out of 50 surveyed homes? Would it be unusual to survey fifty homes and to find that only two of them did not use a land line?

20. A survey found in Harper's Index found that 86% of Americans have never been the victim of a violent crime. If researchers plan to survey randomly selected groups of 20 people, what is the expected (average) number and standard deviation of people they will encounter who have been a victim of a violent crime? Would it be unusual to find a group that has 9 people in it who have been victims of violent crime?

21. Seventy-five percent of youths ages 12 – 17 have systolic blood pressures under 136 mm of mercury. If we survey batches of 30 youths ages 12 – 17, what will the average and standard deviation be for the number of youths under 136 mm for their systolic bp measure? Would it be unusual to find a group where only half of them have values under 136?

22. 51.5% of women aged 20-34 are overweight in the US 1999-2000 (Health United States, 2003, NCHS). A researcher plans to conduct a phone survey of 50 women aged 20 – 34 and as part of the survey ask about their height, weight, and exercise habits. What is the average number of women out of 50 that will indicate they are overweight? What is the standard deviation? Would it be unusual to find that only 10 women report information that indicates they are overweight? If this happens what are we likely to believe?

23. 58% of men aged 20-34 are overweight in the US 1999-2000 (Health United States, 2003, NCHS). If we study groups of 100 men what will the mean and standard deviation be for the number of overweight men in groups of 100 men aged 20 – 34. Would it be unusual to find that only 42 of them are overweight?