

# Some Hypothesis Testing Examples

## One Tailed (Upper Tailed)

An insurance company is reviewing its current policy rates. When originally setting the rates they believed that the average claim amount was \$1,800. They are concerned that the true mean is actually higher than this, because they could potentially lose a lot of money. They randomly select 40 claims, and calculate a sample mean of \$1,950. Assuming that the standard deviation of claims is \$500, and set  $\alpha = .05$ , test to see if the insurance company should be concerned.

### *Solution*

- Step 1: Set the null and alternative hypotheses

$$H_0 : \mu \leq 1800$$

$$H_1 : \mu > 1800$$

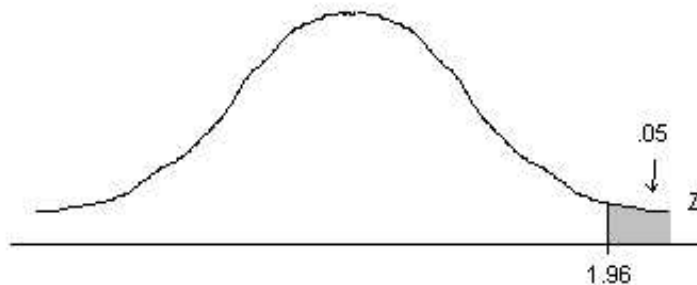
- Step 2: Calculate the test statistic

$$Z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} = \frac{1950 - 1800}{500/\sqrt{40}} = 1.897$$

- Step 3: Set Rejection Region

Looking at the the picture below, we need to put all of  $\alpha$  in the right tail. Thus,

$$R : Z > 1.96$$



- Step 4: Conclude

We can see that  $1.897 < 1.96$ , thus our test statistic is not in the rejection region. Therefore we fail to reject the null hypothesis. We cannot conclude anything statistically significant from this test, and cannot tell the insurance company whether or not they should be concerned about their current policies.

### One Tailed (Lower Tailed)

Trying to encourage people to stop driving to campus, the university claims that on average it takes people 30 minutes to find a parking space on campus. I don't think it takes so long to find a spot. In fact I have a sample of the last five times I drove to campus, and I calculated  $\bar{x} = 20$ . Assuming that the time it takes to find a parking spot is normal, and that  $\sigma = 6$  minutes, then perform a hypothesis test with level  $\alpha = .10$  to see if my claim is correct.

*Solution*

- Step 1: Set the null and alternative hypotheses

$$H_0 : \mu \geq 30$$

$$H_1 : \mu < 30$$

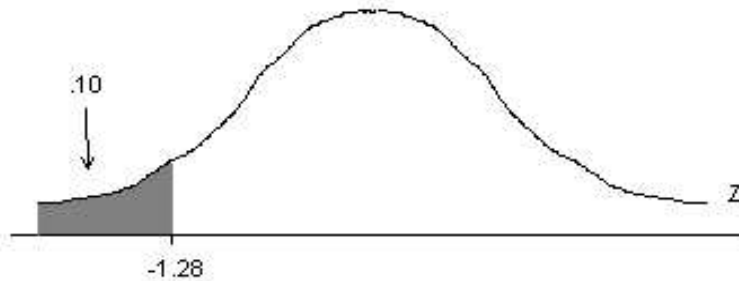
- Step 2: Calculate the test statistic

$$Z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} = \frac{20 - 30}{6/\sqrt{5}} = -3.727$$

- Step 3: Set Rejection Region

Looking at the the picture below, we need to put all of  $\alpha$  in the left tail. Thus,

$$R : Z < -1.28$$



- Step 4: Conclude

We can see that  $-3.727 < -1.28$ , thus our test statistic is in the rejection region. Therefore we reject the null hypothesis in favor of the alternative. We can conclude that the mean is significantly less than 30, thus I have proven that the mean time to find a parking space is less than 30.

### Two Tailed

A sample of 40 sales receipts from a grocery store has  $\bar{x} = \$137$  and  $\sigma = \$30.2$ . Use these values to test whether or not the mean is sales at the grocery store are different from \$150.

*Solution*

- Step 1: Set the null and alternative hypotheses

$$H_0 : \mu = 150$$

$$H_1 : \mu \neq 150$$

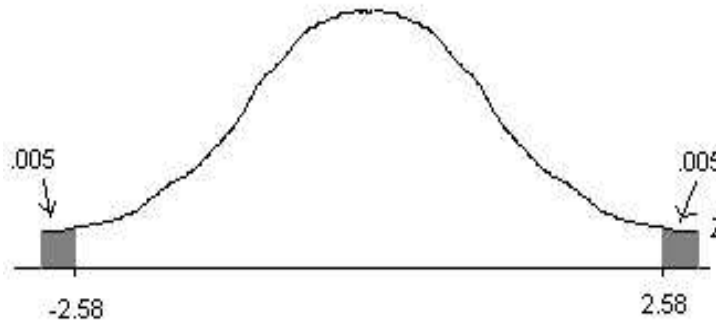
- Step 2: Calculate the test statistic

$$Z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} = \frac{137 - 150}{30.2/\sqrt{40}} = -2.722$$

- Step 3: Set Rejection Region

Looking at the the picture below, we need to put half of  $\alpha$  in the left tail, and the other half of  $\alpha$  in the right tail. Thus,

$$R : |Z| > 2.58$$



- Step 4: Conclude

We can see that  $|-2.722| = 2.722 > 2.58$ , thus our test statistic is in the rejection region. Therefore we reject the null hypothesis in favor of the alternative. We can conclude that the mean is significantly different from \$150, thus I have proven that the mean sales at the grocery store is not \$150.