The **Poisson distribution** is a discrete distribution. It is often used as a model for the number of events (such as the number of telephone calls at a business, number of customers in waiting lines, number of defects in a given surface area, airplane arrivals, or the number of accidents at an intersection) in a specific time period. It is also useful in ecological studies, e.g., to model the number of prairie dogs found in a square mile of prairie. The major difference between Poisson and Binomial distributions is that the Poisson does not have a fixed number of trials. Instead, it uses the fixed interval of time or space in which the number of successes is recorded.

**Parameters: The mean is λ. The variance is λ.**

1. Let $X$ equal the number of typos on a printed page. (This is an example of an interval of space — the space being the printed page.)
2. Let $X$ equal the number of cars passing through the intersection of Allen Street and College Avenue in one minute. (This is an example of an interval of time — the time being one minute.)
3. Let $X$ equal the number of Alaskan salmon caught in a squid driftnet. (This is again an example of an interval of space — the space being the squid driftnet.)
4. Let $X$ equal the number of customers at an ATM in 10-minute intervals.
5. Let $X$ equal the number of students arriving during office hours.

**Poisson Random Variable**

If $X$ is a Poisson random variable, then the probability mass function is:

$$f(x) = \frac{e^{-\lambda}\lambda^x}{x!}$$

for $x = 0, 1, 2, \ldots$ and $\lambda > 0$, where $\lambda$ will be shown later to be both the mean and the variance of $X$.

$$p(x, \lambda) = \frac{e^{-\lambda}\lambda^x}{x!} \qquad \text{for } x = 0, 1, 2, \cdots$$

$\lambda$ is the parameter which indicates the average number of events in the given time interval.

## A Poisson Process meets the following criteria:

1. Events are **independent** of each other. The occurrence of one event does not affect the probability another event will occur.
2. The average rate (events per time period) is constant.
3. Two events cannot occur at the same time.

**Ex.1. On an average Friday, a waitress gets no tip from 5 customers. Find the probability that she will get no tip from 7 customers this Friday.**

The waitress averages 5 customers that leave no tip on Fridays: $\lambda = 5$.

Random Variable : The number of customers that leave her no tip this Friday.

We are interested in P(X = 7).

**Ex. 2 During a typical football game, a coach can expect 3.2 injuries. Find the probability that the team will have at most 1 injury in this game.**

A coach can expect 3.2 injuries : $\lambda = 3.2$.

Random Variable : The number of injuries the team has in this game.

We are interested in $P(X \leq 1)$.

**Ex. 3**. **A small life insurance company has determined that on the average it receives 6 death claims per day. Find the probability that the company receives at least seven death claims on a randomly selected day.**

$$P(x \geq 7) = 1 - P(x \leq 6) = 0.393697$$

**Ex. 4**. **The number of traffic accidents that occurs on a particular stretch of road during a month follows a Poisson distribution with a mean of 9.4. Find the probability that less than two accidents will occur on this stretch of road during a randomly selected month.**

$$P(x < 2) = P(x = 0) + P(x = 1) = 0.000860$$

## Poisson distribution examples

1. The number of road construction projects that take place at any one time in a certain city follows a Poisson distribution with a mean of 3. Find the probability that exactly five road construction projects are currently taking place in this city. (0.100819)

2. The number of road construction projects that take place at any one time in a certain city follows a Poisson distribution with a mean of 7. Find the probability that more than four road construction projects are currently taking place in the city. (0.827008)

3. The number of traffic accidents that occur on a particular stretch of road during a month follows a Poisson distribution with a mean of 7.6. Find the probability that less than three accidents will occur next month on this stretch of road. (0.018757)

4. The number of traffic accidents that occur on a particular stretch of road during a month follows a Poisson distribution with a mean of 7. Find the probability of observing exactly three accidents on this stretch of road next month. (0.052129)

5. The number of traffic accidents that occur on a particular stretch of road during a month follows a Poisson distribution with a mean of 6.8. Find the probability that the next two months will both result in four accidents each occurring on this stretch of road. (0.009846)

6. Suppose the number of babies born during an 8–hour shift at a hospital's maternity wing follows a Poisson distribution with a mean of 6 an hour. Find the probability that five babies are born during a particular 1–hour period in this maternity wing. (0.160623)

7. The university policy department must write, on average, five tickets per day to keep department revenues at budgeted levels. Suppose the number of tickets written per day follows a Poisson distribution with a mean of 8.8 tickets per day. Find the probability that less than six tickets are written on a randomly selected day from this distribution. (0.128387)

8. The number of goals scored at State College hockey games follows a Poisson distribution with a mean of 3 goals per game. Find the probability that each of four randomly selected State College hockey games resulted in six goals being scored. (.00000546)