

Probability Introduction

An **experiment** is an act of observation that leads to a single outcome that cannot be predicted with certainty.

An **event** is a specific collection of sample points.

A **sample point (or simple event)** is the most basic outcome of an experiment.

Sample point ~ outcome

A **sample space** (denoted S) is the collection of all possible outcomes of an experiment.

Examples:

List the different possible families that can occur when a couple has three children...

List the possible outcomes for three flips of a fair coin...

The **probability of an event A** is calculated by summing the probabilities of the sample points in the sample space for A.

$P(A) = \frac{n}{N}$, n = # of times you observed event A (number of ways A can happen), N = number of observations (Number of total possibilities).

Example: Using the sample space created above for having three children, find the probability of having at least two girls.

Probability Rules for Sample Points:

1. All sample point probabilities must be between 0 and 1.
2. The sum of the individual sample point probabilities must be equal to 1.

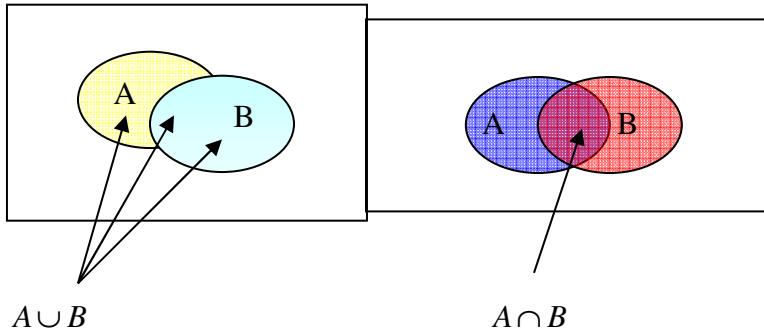
Example: Use the data summarized below to determine the probability that a subject did not use marijuana.

Also, find the probability of a false – positive.

	Did the subject use marijuana?		Totals
	Yes	No	
Positive test result	119	24	143
Negative test result	3	154	157
Total	122	178	300

The **Union** of two events is the event that A or B or Both occur on a single performance of an experiment. We use the notation: $A \cup B$

The **Intersection** of two events A and B is the event that occurs if both A and B occur on a single performance of the experiment. We use the notation: $A \cap B$



Two events are **mutually exclusive** if they cannot occur together at the same time.

If two sets are mutually exclusive:

1. $P(AB) = 0$
2. $P(A \cup B) = P(A) + P(B)$

If two sets are not mutually exclusive: $P(A \cup B) = P(A) + P(B) - P(AB)$

The **Complement** of a given set is all the events belonging to the sample space S, that do not belong to the given set.

$$\bar{A} = A \text{ Complement}$$

Some important logical conclusions regarding complements (try to think about why these are true):

$$\bar{A} \cup A = S, P(A) + P(\bar{A}) = 1, P(A) = 1 - P(\bar{A})$$

Example: If the probability of losing money on a business adventure is 20%, the probability of making money is 35%, and the probability of breaking even is 45%, what is the probability of not losing money?

An important rule of probability is the **probability of “at least one”**. The rule is as follows: $P(\text{at least one}) = 1 - P(\text{none})$

Example: Find the probability of awaking in the morning when using three alarm clocks if each alarm clock has a 99% chance of waking you.

Addition Rule of Probability: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Example: Find the probability of randomly selecting one card from a deck of 52 cards and getting either a face card or a heart.

Recall: If A and B are mutually exclusive events, $P(A \cap B) = 0$.

Example: Find the probability of randomly selecting one card from a deck of 52 cards and getting either a face card or a three.

Pregnancy Test Results			
	Positive result (Preg. Indicated)	Negative Result (Preg. Not Indicated)	Totals
Subject is pregnant	80	5	85
Subject is not Pregnant	3	11	14
Totals	83	16	99

Example: (Use the table above) Find the probability of a single randomly selected subject being pregnant or having a positive test result.

Find the probability of her having a false positive or a false negative exam result.

Conditional Probability Formula: To find the probability that event A occurs given event B occurs, we divide the probability that both event A and B occur by the probability that B occurs,

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Example: Find the probability of a subjecting having a false positive result given that the subject has a positive result.

Example: Find the probability of a subject getting a positive result given that the subject does smoke marijuana. Do the problem again given the person does not smoke marijuana?

	Did the subject use marijuana?		Totals
	Yes	No	
Positive test result	119	24	143
Negative test result	3	154	157
Total	122	178	300

Example: Find the probability of a person having a positive result on a pregnancy test given that they are not pregnant. Then find the probability of a person not being pregnant given that they have a positive pregnancy test. Are these probabilities the same?

Pregnancy Test Results			
	Positive result	Negative Result	
	(Preg. Indicated)	(Preg. Not Indicated)	Totals
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Independence: We can use the conditional probability formula to define independence: Two events are independent iff (if and only if),

$$P(A|B) = P(A) \text{ And } P(B|A) = P(B)$$

This result tells us that the two rolls of the die are independent events.

Multiplication Rule for Probability: The probability that two events A and B occur is given by:

$$P(A \text{ and } B) = P(A \cap B) = P(A) \bullet P(B|A)$$

If the two events A and B are independent, we may use:

$$P(A \text{ and } B) = P(A \cap B) = P(A) \bullet P(B)$$

Example: Find the probability of a guessing the correct answers on three different multiple choice questions that have 5 answer choices each.

Example: If a store has 20 DVD players on a shelf that work perfectly and 3 that are defective, find the probability of randomly selecting two defective DVD players when purchasing a pair of DVD players from the store.

Example: In order for a container of Sony HDTV sets to be offloaded from a truck for sale at a retail store, a random selection of three sets is inspected from the container. If the three sets are defect free, the entire container will be accepted without further inspection. If an individual Sony HDTV has a 0.2% defective rate, what is the probability that a container of Sony HDTV sets will require further inspection?