

**1. The tread life of a particular brand of tire is a random variable best described by a normal distribution with a mean of 60,000 miles and a standard deviation of 8,300 miles.**

a) What proportion of the tires of this particular brand last **longer than 54,000** miles?

$$Z = \frac{54000 - 60000}{8300} = -0.72 \quad P = 0.5 + 0.2642 = 0.7642$$

b) What proportion of the tires last **between 40,000 and 45,000** miles?

$$Z_1 = \frac{40000 - 60000}{8300} = -2.40 \quad Z_2 = \frac{45000 - 60000}{8300} = -1.80 \quad P = 0.4918 - 0.4641 = 0.0277$$

c) Find the point in the mileage distribution, which **33% of the tires will exceed**.

$$X_0 = \mu + z\sigma = 60,000 + 0.44 \times 8,300 = 60,365 \text{ mi}$$

d) Suppose the manufacturer guarantees the tread life of the tires for **the first 55,000** miles. What proportion of the tires will need to be replaced under warranty?

$$Z = \frac{55000 - 60000}{8300} = -0.60 \quad P = 0.5 - 0.2257 = 0.2743$$

e) How long should tires be guaranteed if the company wishes to replace **less than 10%** of the tires?

$$X_0 = \mu + z\sigma = 60,000 + (-1.28 \times 8,300) = 49,376 \text{ mi}$$

g) What warranty should the company use if they want **96%** of the tires to outlast the warranty?

$$X_0 = \mu + z\sigma = 60,000 + (-1.75 \times 8,300) = 45,475 \text{ ml}$$

**2. The heights of women are normally distributed with a mean of 63.6 inches and a standard deviation of 2.5 inches. If 16 women are selected at random, find the mean and standard deviation of the population of sample means.**

$$\mu_{\bar{x}} = \mu = 63.6 \quad \sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{2.5}{\sqrt{16}} = 0.625$$

3. As part of a quality control program at a factory, random samples of 36 6-ounce cans of juice are taken, and the contents carefully measured. When the manufacturing process is working properly, the cans are filled with an average of 6.04 oz of juice. The standard deviation is 0.02 oz. When the process is working properly, what is the probability a sample of 36 of the 6-oz cans of juice would contain an average of 6.03 oz or less?

$$Z = \frac{6.04 - 6.03}{\frac{0.02}{\sqrt{36}}} = -3.00$$

$$P = 0.5 - 0.4987 = 0.0013$$

4. The weight of corn chips dispensed into a 10-ounce bag by the dispensing machine has been identified as possessing a normal distribution with a mean of 10.5 ounces and a standard deviation of .2 ounces. Suppose 100 bags of chips were randomly selected from this dispensing machine. Find the probability that the sample mean weight of these 100 bags exceeded 10.45 ounces.

$$Z = \frac{10.45 - 10.5}{\frac{0.2}{\sqrt{100}}} = -2.50$$

$$P = 0.5 + 0.4938 = 0.9938$$

5. A professor believes that if a class is allowed to work on an examination as long as desired, the times spent by the students would be best described by a normal distribution with a mean of 40 minutes and a standard deviation of 6 minutes. Approximately how long should be allotted for the examination if the professor wants almost all, say 98%, of the class to finish?

$$X_0 = \mu + z\sigma = 40 + 2.05 \times 6 = 52.3 \text{ min}$$

6. Consider the discrete probability distribution when answering the following question.

<b>X</b>	2	4	5	10
<b>P(X)</b>	.2	.3	?	.2

- Find the probability that **X is large than 2**.
- Calculate the **mean** and **variance** for this distribution.
- Find the probability that **X is at most 4**.
- Find the probability that  $\mu - \sigma < X < \mu + \sigma$ .

Answer:

$$P(x > 2) = [P(x=4) + P(x=5) + P(x=10)] = (.3 + .3 + .2) = .8$$

$$\mu = \sum xp(x) = 2(.2) + 4(.3) + 5(.3) + 10(.2) = 5.1$$

$$\sigma^2 = \sum x^2p(x) - \mu^2 = 4(.2) + 16(.3) + 25(.3) + 100(.2) - (5.1)^2 = 33.1 - 26.01 = 7.09$$

$$\sigma = 2.66$$

$$P(X \leq 4) = P(x=2) + P(x=4) = .5$$

$$\mu - \sigma = 5.1 - 2.66 = 2.44$$

$$\mu + \sigma = 5.1 + 2.66 = 7.76$$

$$P(2.44 < X < 7.76) = P(x=4) + P(x=5) = .6$$

**7. Current estimates suggest that in east Africa almost 60% of the households have a PC with access to on-line services. Suppose 20 people with home-based PC were randomly and independently sampled.**

a) Find the probability that **at least 10 but at most 14** currently have access to on-line services.

$$P(10 \leq x \leq 14) = P(x \leq 14) - P(x \leq 9) \quad \text{Use } n = 20, p = .6$$

b) Find the probability that **fewer than quarter** of those sampled have **no access** to on-line services.

$$P(x < 5) = P(x \leq 4) \quad \text{Use } n = 20, p = 1 - .6 = .4$$

c) Find the probability that **at least 18** currently have access to on-line services.

$$P(x \geq 18) = 1 - P(x \leq 17) \quad \text{Use } n = 20, p = .6$$

d) How many people **do you expect** currently **do not** have access to on-line services?

$$E(x) = \mu = np = 20 \times .4 = 8$$

**8. According to a recent study, 1 in every 4 women has been a victim of domestic abuse at some point in their lives. Suppose we have randomly and independently sampled twenty women. Find the probability that at least 2 of the sampled women have been the victim of domestic abuse at some point in their lives.**

**n** = 20 [the total number of women]

**k** = 2 [the number of women you are looking for]

**p** = .25 [the probability that they are victims]

**q** = .75 [the probability that they are not victims]

$$P(x \geq 2) = 1 - P(x < 2) = 1 - [P(x = 0) + P(x = 1)] = 0.976$$

$$P_{(1)} = \frac{20!}{1!(20-1)!} (.25^1)(.75^{20-1}) = 0.021$$

$$P_{(0)} = \frac{20!}{1!(20)!} (.25^0)(.75^{20}) = 0.003$$

**9. In analyzing hits by V-1 buzz bombs in World War II, Moscow was subdivided into 600 regions, each with an area of 0.35 km<sup>2</sup>. A total number of 1200 bombs hit the combined area of 600 regions. (Use a Poisson Distribution)**

1) What is the average number of bombs that hit the randomly selected region?

$$\lambda = \frac{1200}{600} = 2$$

2) What is the corresponding variance?

$$\sigma^2 = 2$$

3) If a region is randomly selected, what is the probability that it was not hit?

$$P(X = 0) = \frac{e^{-\lambda}}{x!} \lambda^x = e^{-2} = 0.135$$

**10. A small life insurance company has determined that on the average it receives 6 death claims per day. Find the probability that the company receives at least seven death claims on a randomly selected day. (Use a Poisson Distribution)**

Answer:  $P(x \geq 7) = 1 - P(x \leq 6) = 0.394$