

## Mathematical Economics Midterm #1, October 3, 2002

You have until 4:45 to complete this exam. Answer all five questions. Each question is worth 20 points, for a total of 100 points. Good luck!

1. Consider the vector  $\mathbf{x} = \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}$ . Find the co-ordinates of  $\mathbf{x}$  in the basis

$$\mathcal{B} = \{\mathbf{b}_1, \mathbf{b}_2, \mathbf{b}_3\} = \left\{ \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \right\}.$$

2. Let

$$A = \begin{pmatrix} 1 & 6 & -7 & 3 \\ 1 & 9 & -6 & 4 \\ 1 & 3 & -8 & 4 \end{pmatrix}.$$

- a) What is the rank of  $A$ ?
- b) Does the equation  $A\mathbf{x} = \mathbf{b}$  have a solution  $\mathbf{x} \in \mathbb{R}^4$  for every  $\mathbf{b} \in \mathbb{R}^3$ ?
- c) If  $A\mathbf{x} = \mathbf{b}$  can be solved, how many solutions does it have?
3. The stationary distribution of the Markov employment model obeys

$$(q - 1)x + py = 0$$

$$(1 - q)x - py = 0$$

$$x + y = 1$$

where  $0 < p < 1$  and  $0 < q < 1$ . Does this system have a solution  $(x, y)$ ? If so, is there a unique solution or multiple solutions? Finally, does the system have a non-negative solution?

4. Find the determinant of the matrix

$$\begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 4 & 9 & 16 \\ 1 & 8 & 27 & 64 \\ 1 & 16 & 81 & 256 \end{pmatrix}$$

A clever solution will save you much time on this one.

5. Consider a modified version of the Keynesian model. We use a standard IS-curve:  $sY + ar = I^o + G$ , but modify the LM curve to depend on the price level  $P$ , so  $M_s = M^o + mY - hr + gP$ . We close the model by adding an upward sloping aggregate supply (AS) curve  $Y = Y^o + bP$ .

- a) Why are the IS and LM curves insufficient to determine  $Y$ ,  $r$ , and  $P$ ?
- b) Show that the model has a unique solution  $(Y, r, P)$ .