

Mathematical Economics Exam #1, September 26, 2011

You have until 2:50 to complete this exam. Answer all four questions. Each question is worth 25 points, for a total of 100 points. Good luck!

1. In \mathbb{R}^3 ,

a) Find all vectors that are perpendicular to

$$\mathbf{x}_1 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \text{ and } \mathbf{x}_2 = \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}.$$

b) Do the vectors described in part (a) form a vector subspace of \mathbb{R}^3 ? Justify your answer.

2. Consider the linear system

$$\begin{aligned} w + x + y + z &= 1 \\ w - x + y + z &= 1 \\ 2w - 3x + 2y + 2z &= 2. \end{aligned}$$

a) What is the rank of the matrix of coefficients?

b) Does this system have any solutions?

c) If the system has one or more solutions, describe them. Is the solution unique?

3. Consider the set $T = \{\mathbf{x} \in \mathbb{R}^3 : \|\mathbf{x}\| = 1\} \subset \mathbb{R}^3$.

a) Is the set T open, closed, or neither when considered as a subset of \mathbb{R}^3 ?

b) Prove your answer in part (a).

4. Consider the following set of vectors in \mathbb{R}^3 :

$$\mathcal{B} = \left\{ \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}, \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix} \right\}.$$

a) Is \mathcal{B} a linearly independent set? Justify your answer.

b) Does \mathcal{B} span \mathbb{R}^3 ? Justify your answer.

c) Is \mathcal{B} a basis for \mathbb{R}^3 ?