Mathematical Economics Final, December 8, 2015

You have until 6:45 to complete this exam. Answer all five questions. **Be sure to justify your answers!** Each question is worth 20 points, for a total of 100 points. Good luck!

1. Consider the differential equation $\ddot{y} - 2\dot{y} + y = 0$. Find the general solution. Then find the solution that obeys $y(0) = 1$, $\dot{y}(0) = 2$.
2. Minimize the function $u(x, y) = x^2 + y^2$ subject to the constraints $x + y^2 = 3$. Don't forget to check the second-order conditions.
3. Consider the differential system

$$\dot{\mathbf{x}} = \begin{pmatrix} -1 & 1 \\ -1 & -1 \end{pmatrix} \mathbf{x}.$$

   a) Find and solve the characteristic equation.

   b) Is $(0, 0)$ an asymptotically stable steady state? Explain.

4. Consider the problem of maximizing the utility function $u(x, y) = x^4 + y^2$ subject to the budget constraint $x + 2y \leq 10$ and the non-negativity constraints $x \geq 0$, $y \geq 0$.

   a) Does this problem have a solution? Explain?

   b) If the problem has a solution, use the Kuhn-Tucker theorem to find it.

5. Is the function $f(x, y, z) = (x^2 + 2xy + zy + z^2)/(x^3 + xyz + 5yz)$ homothetic? Explain.