Mathematical Economics Final, December 6, 2016

You have until 6:45 to complete this exam. Answer all five questions. **Be sure to justify your answers!** Each question is worth 20 points, for a total of 100 points. Good luck!

1. Consider the differential equation $\ddot{y} + 9y = 0$. Find the general solution. Then find the solution that obeys $y(0) = 1$, $\dot{y}(0) = 2$.
2. Minimize the function $u(x, y) = (x - 1)^2 + (y - 2)^2$ subject to the constraint $x + 3y \leq 3$. Don't forget to check the second-order conditions and constraint qualification.
3. Is the function $f(x, y) = x^2 + y^2$ quasiconcave and/or quasiconvex on $\mathbb{R}^2_{++}$? Explain.
4. Consider the difference equation

$$\mathbf{x}_{n+1} = \begin{pmatrix} 1 & 2 \\ 3 & -4 \end{pmatrix} \mathbf{x}_n.$$

   a) Find the eigenvalues of the system.

   b) Find eigenvectors corresponding to the eigenvalues.

   c) Do the eigenvectors form a basis for $\mathbb{R}^2$? Explain.

5. Consider the problem of maximizing the function $u(x, y) = x^{1/2} + 3y$ subject to the constraint $x + 2y \leq 10$ and the non-negativity constraints $x \geq 0$, $y \geq 0$.

   a) Does this problem have a solution? Explain?

   b) If the problem has a solution, use the Kuhn-Tucker theorem to find it. Don't forget to check constraint qualification and the second-order conditions.