Homework Assignment #2

8.3 Show that if AB is defined, then $B^\dagger A^\dagger$ is defined but $A^\dagger B^\dagger$ need not be defined.

Answer: Let A be $K \times M$ and B be $M' \times N$. We must have $M = M'$ for the product AB to be defined. In that case, $B^\dagger$ is $N \times M$ and $A^\dagger$ is $M \times K$, so the product $B^\dagger A^\dagger$ is defined and is $N \times K$.

For $A^\dagger B^\dagger$ to be defined, we must have $K = N$, which may not happen.

8.18 Show by simple matrix multiplication that, if $ad - bc \neq 0$,

$$\frac{1}{ad-bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

is both a left and right inverse of $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$.

Answer:

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \times \frac{1}{ad-bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix} = \frac{1}{ad-bc}\begin{pmatrix} ad-bc & 0 \\ 0 & ad-bc \end{pmatrix} = I_2$$

and

$$\frac{1}{ad-bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix} \times \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \frac{1}{ad-bc}\begin{pmatrix} ad-bc & 0 \\ 0 & ad-bc \end{pmatrix} = I_2.$$

9.8: Use the observation following Theorem 9.2 to carry out a quick calculation of the determinant of each of the following matrices:

$$a)\begin{pmatrix} 1 & 1 & 1 \\ 1 & 4 & 2 \\ 1 & 4 & 3 \end{pmatrix} \qquad b)\begin{pmatrix} 1 & 1 & 1 \\ 0 & 4 & 5 \\ 1 & 9 & 6 \end{pmatrix}.$$

Answer: We row reduce the first matrix by first subtracting the first row from the second and third rows, obtaining $\begin{pmatrix} 1 & 1 & 1 \\ 0 & 3 & 1 \\ 0 & 3 & 2 \end{pmatrix}$, and then subtract the second row from the first, obtaining $\begin{pmatrix} 1 & 1 & 1 \\ 0 & 3 & 1 \\ 0 & 0 & 1 \end{pmatrix}$. As with all row-echelon matrices, we now find the determinant by multiplying the diagonal terms. Thus the matrix in (a) has determinant 3.

For the second matrix, first subtract row 1 from row 3, and then subtract twice row 2 from row 3, obtaining $\begin{pmatrix} 1 & 1 & 1 \\ 0 & 4 & 5 \\ 0 & 0 & -5 \end{pmatrix}$. This has determinant -20.

9.16 If you are familiar with partial derivatives, compute

$$\frac{\partial Y}{\partial a} = \frac{-rh}{(sh+am)} \leq 0.$$

So an increase in the marginal efficiency of capital a will bring down the equilibrium Y and r. How will the equilibrium Y change if h increases? How will the equilibrium r change if m or s increases?

Answer: From Section 9.3,

$$Y = \frac{(I^o+G)h + a(M_s - M^o)}{sh+am}, \qquad r = \frac{(I^o+G)m - s(M_s - M_o)}{sh+am}.$$

Now

$$\frac{\partial Y}{\partial a} = \frac{1}{(sh+am)^2}[sh(M_s - M^o) - mh(I^o+G)] = \frac{-rh}{(sh+am)}.$$

A similar calculation shows

$$\frac{\partial Y}{\partial h} = \frac{ra}{(sh + am)} \geq 0,$$

so equilibrium Y increases when h increases. The other calculations are also similar, yielding

$$\frac{\partial r}{\partial m} = \frac{sY}{(sh + am)} \geq 0, \text{ and } \frac{\partial r}{\partial s} = \frac{-mY}{(sh + am)} \leq 0.$$

Thus r increases when m increases and decreases when s increases.

26.14 Find the exact values of k which make each of the following matrices singular:

$$a) \begin{pmatrix} 1 & k \\ k & 1 \end{pmatrix} \qquad b) \begin{pmatrix} k & 1 & 1 \\ 1 & k & 1 \\ 1 & 1 & k \end{pmatrix}.$$

Answer:

a) In this case, the determinant is $1 - k^2$, so the matrix will be singular if and only if $k^2 = 1$, which corresponds to $k = \pm 1$.

b) Here the determinant is $k^3 - 3k + 2$, which factors to $(k-1)(k^2 + k - 2) = (k-1)^2(k-2)$. The matrix is singular if and only if $k \in \{1, -2\}$.