Mathematical Economics Final, December 4, 2018

You have until 6:50 to complete this exam. Answer all five questions. **Be sure to justify your answers!** Each question is worth 20 points, for a total of 100 points. Good luck!

1. On $\mathbb{R}^2$, maximize $3x + y$ under the constraint $x^2 + y \leq 9$. Note that there are no non-negativity constraints. Don't forget to check constraint qualification. What do the second order conditions tell you? Is your solution a maximum?
2. On $\mathbb{R}^2_+$, find demand by maximizing the utility function $u(x, y) = 3x - e^{-y}$ subject to the constraints $p_x x + p_y y \leq m$, $x \geq 0$, $y \geq 0$ where $p_x, p_y, m > 0$. Don't forget to check constraint qualification and the second order conditions.
3. Consider the differential equation $\ddot{y} - 4\dot{y} + 3y = t + 1$ with initial conditions $y(0) = 7/3$ and $\dot{y}(0) = 4/3$.
   a) Find the general solution of the associated homogeneous equation.
   b) Find a particular solution of the inhomogeneous equation.
   c) Find the solution that obeys the initial conditions.
4. Let $S = \{(x_1, x_2, x_3) : x_1^2 + x_2^2 + x_3^2 = 1\}$ be the unit sphere in $\mathbb{R}^3$.
   a) Use the Implicit Function Theorem to show that if $\mathbf{x}^0 = (x_1^0, x_2^0, x_3^0) \in S$, we can write one of the coordinates in terms of the other two in some neighborhood of $\mathbf{x}^0$.
   b) Give an example of how to do this at the point $\mathbf{x}^0 = (0, 1, 0)$.
5. Consider the difference equation $\mathbf{x}_{t+1} = \mathbf{A}\mathbf{x}_t$ where

$$\mathbf{A} = \begin{pmatrix} 0 & -1 \\ -1 & 3/2 \end{pmatrix}.$$

   a) Find the eigenvalues.
   b) Find a non-zero vector $\mathbf{x}$, so that if $\mathbf{x}_0 = \mathbf{x}$, then $\mathbf{x}_t \to \mathbf{0}$.