# Mathematical Economics Exam #1, October 1, 2019

You have until 6:15 to complete this exam. Answer all four questions. **Be sure to justify your answers!** Each question is worth 25 points, for a total of 100 points. Good luck!

1. Consider the set
$$\mathscr{B} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 4 \end{pmatrix}$$

   *a*) Is $\mathscr{B}$ a basis?

   *b*) If $\mathscr{B}$ is a basis, find the coordinates of the vector $\mathbf{x} = (1,2,3)^{\mathsf{T}}$ in the basis $\mathscr{B}$. If $\mathscr{B}$ is not a basis, find a non-zero vector that is **not** in the span of $\mathscr{B}$.

2. Let $\mathbf{A} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$.

   *a*) Compute the rank of $\mathbf{A}$.

   *b*) What is the dimension of the nullspace (kernel) of $\mathbf{A}$, $\{\mathbf{x} : \mathbf{A}\mathbf{x} = \mathbf{0}\}$?

   *c*) Find a basis for the nullspace of $\mathbf{A}$.

3. Consider the following sets:
$$S = \cup_{n=1}^{\infty}(-n, 0), \qquad T = \cap_{n=1}^{\infty}(-n, +n).$$

   *a*) Is $S$ an open set? Explain

   *b*) Is $T$ open? Closed? Explain.

4. Consider the sequence
$$x_n = \left\{ (-1)^n \frac{n+1}{n+3} \right\}_{n=1}^{\infty}.$$

   Does this sequence converge? If so, find the limit. If not, does it have any cluster points. Explain.