Micro II Final, December 12, 2007

1. A lottery pays \$10 with probability .1 and \$0 with probability .9. The consumer's utility function is $u(c) = c^\eta$ where $0 < \eta < 1$.

   a) Compute the certainty equivalent of this lottery as a function of η.

   Answer: Expected utility is $.1 \times 10^\eta = 10^{\eta-1}$. We find the certainty equivalent $c(\eta)$ by solving $u(c(\eta)) = 10^{\eta-1}$, so $c(\eta) = 10^{1-1/\eta}$.

   b) What happens to the certainty equivalent as $\eta \to 1$?

   Answer: $\lim_{\eta\to 1} c(\eta) = 1$.

   c) What happens to the certainty equivalent as $\eta \to 0$?

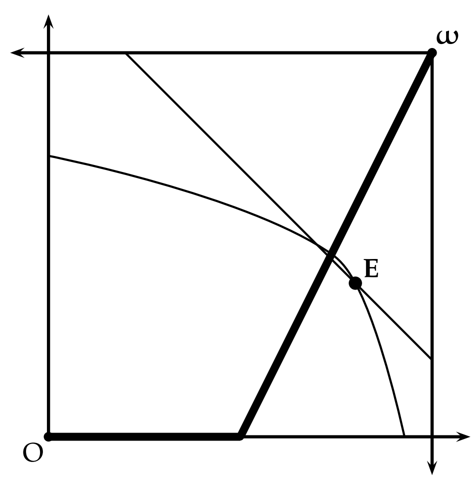
   Answer: $\lim_{\eta\to 0^+} c(\eta) = 0$.

2. Consider a two-person, two-good exchange economy. Consumer 1 has endowment $(4, 2)$ and utility $u_1(x_1, x_2) = x_1 + x_2$. Consumer 2 has endowment $(1, 3)$ and utility $u_2(x_1, x_2) = \ln x_1 + 2 \ln x_2$.

   a) Find all Pareto optima.

   Answer: We can find all interior Pareto optima by setting the marginal rates of substitute equal. This yields $1 = x_2^2/(2x_1^2)$. These Pareto optima obey $2x_1^2 = x_2^2$.

   The total endowment is $(5, 5)$, so we reach the boundary of the Edgeworth box when $x_2^2 = 5$ and $x_1^2 = 2.5$. Consumer one receives $\mathbf{x}^1 = (2.5, 0)$. As we move along the boundary toward $(0, 0)$, consumer 2 is progressively better off while consumer one is worse off. These points comprise the rest of the Pareto optima. This is illustrated by the heavy line in the diagram.



   b) Find the core.

   Answer: In a two-person exchange economy, an allocation is in the core if and only if it is individually rational and Pareto optimal. Individual rationality requires $x_1^1 + x_2^1 = u_1(\mathbf{x}^1) =\geq u_1(\boldsymbol{\omega}^1) = 6$ and $\ln x_1^2 + 2\ln x_2^2 = u_2(\mathbf{x}^2) \geq u_2(\boldsymbol{\omega}^2) = \ln 1 + 2\ln 3 = \ln 9$. Thus $x_1^2(x_2^2)^2 \geq 9$.

   These conditions cannot be satisfied at any boundary Pareto optima, so we must be in the interior. The conditions can be expressed in terms of x_1^2. Individual rationality for consumer one is $10 - x_1^2 - x_2^2 \geq 6$. Using $x_2^2 = 2x_1^2$, this becomes $4 \geq 3x_1^2$ or $x_1^2 \leq 4/3$. Individual rationality for consumer two is $x_1^2(2x_1^2)^2 \geq 9$. This can be written as $x_1^2 \geq (3/2)^{2/3}$. Thus $4/2 \geq x_1^2 \geq (3/2)^{2/3}$ characterizes the core (with $x_2^2 = 2x_1^2$, $x_1^1 = 5 - x_1^2$ and $x_2^1 = 5 - 2x_1^2$).

The line and curve through the endowment point E demarcate the individualy rational points. A close look at the diagram shows the thin sliver that is the core.

c) Find all Walrasian equilibria.

Answer: Since consumer two has Cobb-Douglas utility, both goods will have positive prices in equilibrium. Take good 1 as numeraire so $\mathbf{p} = (1, p)$. Consumer incomes are then $m_1 = 4 + 2p$ and $m_2 = 1 + 3p$. Consumer two has demand $m_2(1/3, 2/3p)$.

If $p = 1$, consumer one demands anything with $x_1^1 + x_2^1 = m_1 = 6$ while consumer two demands $\mathbf{x}^2 = (4/3, 8/3)$. In that case $\mathbf{x}^1 = (11/3, 7/3)$ adds to six and also satisfies market clearing. This is an equilibrium.

If $p < 1$, consumer one demands only good 2. Demand for good 1 is then $(1 + 3p)/3$. Setting it equal to supply (5), we find $p = 14/3$, contradicting $p < 1$. There is no equilibrium here.

If $p > 1$, consumer one demands only good 1. Demand for good 2 is $2(1 + 3p)/3p = 5$. So $p = 2/9$, contradicting $p > 1$. Thus $p = 1$ is the only equilibrium price.

3. Suppose a firm's production set is given by $Y = \{(-z, q) : z \geq 0, q \leq z^{1/3}\}$.

a) Find the profit-maximizing net output vector.

Answer: Profit is $p_q z^{1/3} - p_z z$. The first-order condition for profit maximization is $p_q z^{-2/3}/3 = p_z$, so $z = (p_q/3p_z)^{3/2}$ and $q = (p_q/3p_z)^{1/2}$. The net output is $(-(p_q/3p_z)^{3/2}, (p_q/3p_z)^{1/2})$.

b) Derive the profit function $\pi(p_z, p_q)$.

Answer: The maximum profit obtained is then $p_q^{3/2}(3p_z)^{-1/2} - p_z^{3/2} p^{-1/2} 3^{-3/2} = 2p_q^{3/2} p_z^{-1/2}/3^{3/2}$.

c) Does the technology exhibit constant returns to scale? Increasing returns to scale? Decreasing returns to scale?

Answer: The production function is strictly concave, so there are decreasing returns to scale.

4. Consider a two-agent, two-good economy production economy where utility is $u_1(x_1, x_2) = (x_1)^{1/2}(x_2)^{1/2}$ and $u_2(x_1, x_2) = (x_1)^{1/3}(x_2)^{2/3}$. Endowments are $\boldsymbol{\omega}^1 = (3, 0)$ and $\boldsymbol{\omega}^2 = (6, 0)$. There is one firm with production set $Y = \{(y_1, y_2) : y_1 \leq 0, y_2 \leq -y_1\}$. Find the equilibrium prices, equilibrium demands by individuals, and the firm's equilibrium net output.

Answer: Since utility is Cobb-Douglas, the equilibrium prices of both goods must be positive, and the demand for both goods will also be positive. Since the aggregate endowment is $(9, 0)$, good 2 must be produced by the firm. The firm's zero profit condition then implies $p_1 = p_2$. We can normalize prices so $\mathbf{p} = (1, 1)$.

Consumer incomes are $m_1 = 3$ and $m_2 = 6$, so demands are $\mathbf{x}^1 = (3/2, 3/2)$ and $\mathbf{x}^2 = (2, 4)$. Market demand is $\mathbf{x} = (7/2, 11/2)$. Now $\mathbf{x} = \boldsymbol{\omega} + \mathbf{y}$, so production is $\mathbf{y} = (-11/2, 11/2) \in Y$.

5. Consider an exchange economy with 2 consumers, 2 goods, and 2 states of the world. Let $x_{s\ell}^i$ denote consumer i's consumption of good ℓ in state s. Each consumer has utility function

$$u(\mathbf{x}^i) = \frac{1}{2}\left[(x_{11}^i x_{12}^i)^{1/2} + (x_{21}^i x_{22}^i)^{1/2}\right].$$

Consumer 1's endowment is $\boldsymbol{\omega}^1 = \big((1, 2), (2, 1)\big)$. Consumer 2's endowment is $\boldsymbol{\omega}_2^1 = \big((2, 1), (1, 2)\big)$. There are two assets with return matrix

$$R = \begin{pmatrix} 1 & .5 \\ .5 & .25 \end{pmatrix}.$$

a) Is there a complete set of assets?

Answer: No, asset 1 has double the return of asset 2, so the rank of the return matrix is 1, which is less than the number of states (2).

b) Find a Radner equilibrium.

Answer: Since there is only one asset, it cannot be traded in equilibrium. (Recall that consumers can only buy one asset by selling another.)

In each spot market, both consumers have equal-weighted Cobb-Douglas utility, so $\mathbf{x}_s = \mathbf{p}\cdot\boldsymbol{\omega}_s(1/2p_{1s}, 1/2p_{2s})$. It follows that $p_{1s} = p_{2s}$. We normalize so $p_{\ell s} = 1$ for all ℓ and s. In that case each consumer has income 3 in each state, and demand is $\mathbf{x}^1 = \mathbf{x}^2 = \big((3/2, 3/2), (3/2, 3/2)\big)$.

Since $\mathbf{r}^1 = 2\mathbf{r}^2$, $q_1 = 2q_2$. Any positive price will insure that consumers will not demand the asset as they then can't afford it! We normalize so that $\mathbf{q} = (1, 1/2)$, although any scalar multiple will do.

c) Is the Radner equilibrium you found Pareto optimal?

Answer: Even though the markets are incomplete, it is Pareto optimal. We can verify this by computing the marginal rates of substitution, all of which are 1. Thus $\mathrm{MRS}^1_{\ell s,kt} = 1 = \mathrm{MRS}^2_{\ell s,kt}$ for all goods k, ℓ and states s, t. This means we our equilibrium allocation (which is interior) is a Pareto optimum.