Micro II Midterm, February 22, 2011

1. There are two goods. Suppose a consumer has the *convex* utility function $u(x_1, x_2) = x_1^2 + x_2^2$. Let prices be $\mathbf{p} = (p_1, p_2) \gg 0$ and income be $m > 0$. Suppose further that $\bar{u} \geq \mathbf{0}$. If possible for this utility function:

   a) Find the Marshallian (ordinary) demand $\mathbf{x}(\mathbf{p}, m)$.

   Answer: Since preferences are strictly convex, the demand will be at one of the corners (or both).

$$\mathbf{x}(\mathbf{p}, m) = \begin{cases} (m/p_1, 0) & \text{if } p_1 < p_2 \\ \{(m/p_1, 0), (0, m/p_2)\} & \text{if } p_1 = p_2 \\ (0, m/p_2) & \text{if } p_1 > p_2 \end{cases}$$

   b) Compute the indirect utility function $v(\mathbf{p}, m)$.

   Answer: The indirect utility is $v(\mathbf{p}, m) = u(\mathbf{x}(\mathbf{p}, m)) = m^2 \max\{p_1^{-2}, p_2^{-2}\}$.

   c) Find the Hicksian (compensated) demand $\mathbf{h}(\mathbf{p}, \bar{u})$.

   Answer: The minimum expenditure also occurs at the corners. Thus

$$\mathbf{h}(\mathbf{p}, \bar{u}) = \begin{cases} (\bar{u}^{1/2}, 0) & \text{if } p_1 < p_2 \\ \{(\bar{u}^{1/2}, 0), (0, \bar{u}^{1/2})\} & \text{if } p_1 = p_2 \\ (0, \bar{u}^{1/2}) & \text{if } p_1 > p_2 \end{cases}$$

   d) Compute the expenditure function $e(\mathbf{p}, \bar{u})$.

   Answer: The expenditure function is $e(\mathbf{p}, \bar{u}) = \mathbf{p}\cdot\mathbf{h}(\mathbf{p}, \bar{u}) = \bar{u}^{1/2} \min\{p_1, p_2\}$.

2. Suppose that production is described by a production function $f(z_1, z_2) = \sqrt{z_1} + 2\sqrt{z_2}$.

   a) Find the conditional factor demand $\mathbf{z}(\mathbf{w}, q)$ for $\mathbf{w} \gg \mathbf{0}$ and $q > 0$.

   Answer: Form the Lagrangian $\mathcal{L} = w_1 x_1 + w_2 x_2 - \lambda(\sqrt{z_1} + 2\sqrt{z_2} - q) - \mu_1 z_1 - \mu_2 z_2$. This yields first-order conditions $w_1 = \lambda/2\sqrt{z_1} + \mu_1$ and $w_2 = \lambda/\sqrt{z_2} + \mu_2$. Obviously, there are no solutions with either $z_1 = 0$ or $z_2 = 0$. By complementary slackness, $\mu_1 = \mu_2 = 0$. We can eliminate λ, finding that $2w_1\sqrt{z_1} = w_2\sqrt{z_2}$, or $4w_1^2 z_1 = w_2^2 z_2$. It follows that $q = \sqrt{z_1} + 2\sqrt{z_2} = (1 + 4w_1/w_2)\sqrt{z_1}$. Then $z_1(\mathbf{w}, q) = q^2 w_2^2/(4w_1 + w_2)^2$ and $z_2(\mathbf{w}, q) = 4q^2 w_1^2/(4w_1 + w_2)^2$.

   b) Suppose output is good 3. Find the corresponding production set Y.

   Answer: Since goods 1 and 2 are inputs, they will be non-positive. The production set is $Y = \{\mathbf{z} \in \mathbb{R}^3 : z_3 \leq \sqrt{z_1} + 2\sqrt{z_2} \text{ and } z_1 \leq 0,\ z_2 \leq 0\}$.

3. There are two consumers and two goods. Consumer 1 has utility $u_1(\mathbf{x}) = (x_1 x_2)^{1/2}$ and income $w_1 > 0$. Consumer 2 has utility $u_2(\mathbf{x}) = x_1 + 2x_2$ and income $w_2 > 0$. Prices are $\mathbf{p} = (p_1, p_2) \gg \mathbf{0}$.

   a) Find the Marshallian demand functions for each consumer.

   Answer: Consumer 1 has equal weighted Cobb-Douglas utility, yielding demand $\mathbf{x}^1(\mathbf{p}, w_1) = (w_1/2)(p_1^{-1}, p_2^{-1})$. Consumer 2 has linear utility. It is easy to see that demand is:

$$\mathbf{x}^2(\mathbf{p}, w_2) = \begin{cases} (w_2/p_1, 0) & \text{when } p_1 < p_2/2 \\ B & \text{when } p_1 = p_2/2 \\ (0, w_2/p_2) & \text{when } p_1 > p_2/2 \end{cases}$$

   where $B = \{\mathbf{x} \in \mathbb{R}_+^2 : p_1 x_1 + p_2 x_2 = w_2\}$.

b) Compute aggregate demand.

Answer: Aggregate demand is

$$\mathbf{x}(\mathbf{p}, w_1, w_2) = \begin{cases} ((w_1 + 2w_2)/p_1, w_1/2p_2) & \text{when } p_1 < p_2/2 \\ B + (w_1/2p_1, w_1/2p_2) & \text{when } p_1 = p_2/2 \\ (w_1/p_1, (w_1 + 2w_2)/2p_2) & \text{when } p_1 > p_2/2 \end{cases}$$

c) Show by example that aggregate demand cannot be written as a function of the price vector $\mathbf{p}$ and aggregate wealth $w = w_1 + w_2$.

Answer: Let $p_1 < p_2/2$. Then $\mathbf{x}(\mathbf{p}, 1, 0) = (1/2p_1, 1/2p_2)$ and $\mathbf{x}(\mathbf{p}, 0, 1) = (1/p_1, 0)$. Although the aggregate income is the same, the aggregate demand is not the same. This means it cannot be written as a function of the price vector and aggregate income.

4. A consumer has preferences on $\mathbb{R}^3_+$ that are described by the utility function $u(\mathbf{x}) = x_1 + (x_2x_3)^{1/2}$.

a) Are these preferences separable relative to the partition $\mathcal{P} = \big\{1, \{2, 3\}\big\}$?

Answer: Yes, it is (weakly) separable relative to $\mathcal{P} = \big\{1, \{2, 3\}\big\}$. Since $(x_1, x_2, x_3) \succsim (x_1', x_2, x_3)$ if and only if $x_1 \geq x_1'$ it induces an order on $\{1\}$, and since $(x_1, x_2, x_3) \succsim (x_1, x_2', x_3')$ if and only if $x_2x_3 \geq x_2'x_3'$, it induces an order on $\{2, 3\}$.

b) Are these preferences completely separable?

Answer: No. Consider $\text{MRS}_{12} = 2(x_2/x_3)^{1/2}$. Since MRS_{12} is affected by the amount of good 3 consumed, these preferences are not completely separable.