Micro II Final, April 28, 2011

1. Consider a two-person, two-good exchange economy. Consumer i has utility $u_i(\mathbf{x}^i) = \max\{x_1^i, x_2^i\}$. The endowments are $\boldsymbol{\omega}^1 = (3,1)$ and $\boldsymbol{\omega}^2 = (0,2)$.

   a) Find all the equilibria of this economy.

   Answer: Due to strong monotonicity, the equilibrium price vector must be strictly positive. If $p_1 \neq p_2$, both consumers will only consume the cheaper good. If that good is good 1, demand for good 1 is $(3p_1 + 3p_2)/p_1 > 3$, so the market for good 1 cannot clear. A similar argument applies if good 2 is cheaper. Take good 1 as numéraire. It follows that the only possible equilibrium price vector is $\mathbf{p}^* = (1,1)$

   Now the income of consumer 1 is 4 and the income of consumer 2 is 2. Demand by consumer 1 is either $(4,0)$ or $(0,4)$. Neither case is consistent with market clearing. There is no equilibrium.

   The equilibrium existence theorem does not apply here because the preferences are not convex. A similar economy with a continuum of equal numbers of consumers like consumers 1 and 2 would have an equilibrium.

   b) Find all Pareto optima.

   Answer: Suppose $\mathbf{x}^1, \mathbf{x}^2$ is an allocation where both consumers receive a positive quantity of good 1. In that case $x_\ell^i < 3$ for all i and ℓ, so $u_i < 3$. If $x_1^1 > x_2^1$, giving the excess to consumer 2 will not harm consumer 1, and will make consumer 2 better off as it will raise his utility to 3. Similar considerations apply if $x_1^1 \leq x_2^1$. Thus no such allocation is Pareto optimal.

   Further, if one consumer gets all of both goods, giving all of one of the goods to the other consumer is a Pareto improvement. It follows that the only Pareto optimal allocations are $((0,3),(3,0))$ and $((3,0),(0,3))$.

2. Suppose there are two inputs and the cost function is $c(q,\mathbf{w}) = q(w_1 + 3w_2)$.

   a) Find the conditional factor demands.

   Answer: By Shephard's Lemma, $\mathbf{z}(q,\mathbf{w}) = q(1,3)$.

   b) Find the production function.

   Answer: Since the conditional factor demands are independent of factor prices, production must be Leontief. With q produced at the corner of the isoquant, $q(1,3)$, the production function is $f(\mathbf{z}) = \min\{z_1, z_2/3\}$.

3. A firm with cost function $c(q) = q^2$ is uncertain about the price that it can receive for its product.

The price is a random variable over $[0, 10]$ with probability density $\frac{3}{1000}p^2$.

a) Suppose the firm is risk-neutral and calculate expected profit as a function of q.

Answer: Profit at price p is $pq - q^2$. Thus expected profit is

$$\frac{3}{1000}\int_0^{10}(pq - q^2)p^2\,dp = \frac{3}{4000}p^4q|_0^{10} - q^2 = \frac{15}{2}q - q^2.$$

b) What q maximizes expected profit?

Answer: Since expected profit is $\frac{15}{2}q - q^2$, the maximum occurs when $\frac{15}{2} = 2q$. In other words, $q = \frac{15}{4}$ maximizes expected profit.

c) Using the choice of q from part (b), what is the probability that profit will be negative?

Answer: Now profit is $\frac{15}{4}p - (\frac{15}{4})^2$. This is negative if and only if $p < \frac{15}{4}$. It follows that the probability of negative profit is $\frac{3}{1000}\int_1^{15/4} p^2\,dp = \frac{1}{1000}p^3|_0^{15/4} = 15^3/40^3 = (3/8)^3 = 27/512 \approx 0.053$.

4. Consider a production economy with 2 goods and 2 consumers. There is one firm with technology set $Y = \{(y_1, y_2) : y_1 \leq -2y_2, y_2 \leq 0\}$. Consumer one has utility $u_1(\mathbf{x}^1) = x_1^1 + 2x_2^1$ and consumer two has utilty $u_2(\mathbf{x}^2) = \sqrt{x_1^2x_2^2}$. Endowments are $\boldsymbol{\omega}^1 = (0, 2)$ and $\boldsymbol{\omega}^2 = (0, 3)$. Find all equilibrium allocations.

Answer: The firm has profit $p_1(-2y_2) + p_2y_2 = (p_2 - 2p_1)y_2$. Since $y_2 \leq 0$, profit cannot be maximized if $p_2 < 2p_1$ while if $p_2 > 2p_1$, the only maximum is at $y_2 = 0$. Since consumer two will demand both goods, and there is no endowment of good one, the firm must produce something. This requires that $p_1 = 2p_2$. Taking good one as numéraire, the normalized equilibrium price vector is $\mathbf{p}^* = (1, 2)$.

Now consumer 1 has income 4 and consumer 2 has income 6. Consumer 2's demand is $\mathbf{x}_2^* = (3, 3/2)$. Consumer 1 will demand any $(4 - 2x, x)$ such that $0 \leq x \leq 2$.

Since both prices are strictly positive, markets clear with equality. Thus $(3, 3/2) + (4 - 2x, x) = (-2y_2, y_2) + (0, 5)$. This implies that $x = y_2 + 7/2$. Since $0 \leq x \leq 2$, $-7/2 \leq y_2 \leq -7/2$. The equilibrium allocations are $\mathbf{x}_1^* = (4 - 2x^*, x^*)$, $\mathbf{x}_2^* = (3, 3/2)$ and $\mathbf{y}^* = (7 - 2x^*, x^* - 7/2)$.

5. Consider an exchange economy with 2 consumers, 2 goods, and 2 states of the world. Let $x_{s\ell}^i$ denote consumer i's consumption of good ℓ in state s. Each consumer has utility function

$$u(\mathbf{x}^i) = \frac{1}{2}\left[(x_{11}^i x_{12}^i)^{1/3} + (x_{21}^i x_{22}^i)^{1/3}\right]$$

Consumer 1's endowment is $\boldsymbol{\omega}^1 = ((2, 2), (0, 0))$. Consumer 2's endowment is $\boldsymbol{\omega}^2 = ((0, 0), (2, 2))$. There is one asset with return vector $\mathbf{r} = (1, .5)$.

a) Is there a complete set of assets?

Answer: No, since there is one asset and it is non-zero, the return matrix has rank 1. As this is less than the number of states (2), the assets are incomplete.

b) Find a Radner equilibrium.

Answer: Since there is only one asset, it cannot be traded in equilibrium. (Recall that consumers can only buy one asset by selling another.)

In each spot market, both consumers have equal-weighted Cobb-Douglas utility, so $x_s = \mathbf{p}\cdot\boldsymbol{\omega}_s(1/2p_{1s}, 1/2p_{2s})$ where $\boldsymbol{\omega}$ is the aggregate endowment. It follows that $p_{1s} = p_{2s}$. We normalize so $p_{\ell s} = 1$ for all ℓ and s. Note that consumer 1 has no income (and so no consumption) in state 2, while the opposite is true for consumer 2.

It follows that $\mathbf{x}^1 = \boldsymbol{\omega}^1$ and $\mathbf{x}^2 = \boldsymbol{\omega}^2$. No actual trade takes place in either spot market

Finally, any positive price of the asset will insure that consumers will not demand the asset as they then can't afford it! We normalize so that $q = 1$, although any scalar multiple will do.

c) Is the Radner equilibrium you found Pareto optimal?

Answer: The equilibrium is not Pareto optimal. Each consumer has utility level $2^{-1/3}$ in equilibrium. The allocation $\mathbf{x}^1 = \mathbf{x}^2 = \big((1,1),(1,1)\big)$ yields utility 1 for each, which is a Pareto improvement.