# Micro I Midterm, February 23, 2012

1. There are two goods. Suppose a consumer has the *convex* utility function $u(x_1, x_2) = x_1 + x_2^2$ and consumption set $\mathfrak{X} = \mathbb{R}^2_+$. Let prices be $\mathbf{p} = (1, p) \gg 0$ and income be $m > 0$. Suppose further that $\bar{u} \geq \mathbf{0}$. If possible for this utility function:

   *a*) Find the Marshallian (ordinary) demand $\mathbf{x}(\mathbf{p}, m)$.

   **Answer:** The indifference curves are parabolas. Because of the convexity of the utility function, maximum utility will be at a corner. The two possibilities are $(m, 0)$ and $(0, m/p)$, If $p^2 > m$, the first one is better, if $p^2 < m$, the second is better, and if $p^2 = m$, they are tied. It follows that the Marshallian demand is:

   $$\mathbf{x}(p, m) = \begin{cases} (m, 0) & \text{if } p^2 > m \\ \left\{(m, 0), \left(0, \frac{m^2}{p^2}\right)\right\} & \text{if } p^2 = m \\ \left(0, \frac{m}{p}\right) & \text{if } p^2 < m \end{cases}.$$

   *b*) Compute the indirect utility function $v(\mathbf{p}, m)$.

   **Answer:** Indirect utility is $v(p, m) = u(\mathbf{x}(p, m)) = \max\{m, m^2/p^2\}$.

   *c*) Find the Hicksian (compensated) demand $\mathbf{h}(\mathbf{p}, \bar{u})$.

   **Answer:** The convex indifference curves imply that expenditure minization also occurs at a corner. We set $u(\mathbf{x}) = \bar{u}$ to find the corners are $(\bar{u}, 0)$ and $(0, \sqrt{\bar{u}})$. The associated expenditures are $\bar{u}$ and $p\sqrt{\bar{u}}$. We choose the first if $\sqrt{\bar{u}} < p$ and the second if $\sqrt{\bar{u}} > p$. Thus

   $$\mathbf{h}(p, \bar{u}) = \begin{cases} (\bar{u}, 0) & \text{if } \sqrt{\bar{u}} < p \\ \{(\bar{u}, 0), (0, \sqrt{\bar{u}})\} & \text{if } \sqrt{\bar{u}} = p \\ (0, \sqrt{\bar{u}}) & \text{if } \sqrt{\bar{u}} > p \end{cases}.$$

   *d*) Compute the expenditure function $e(\mathbf{p}, \bar{u})$.

   **Answer:** Expenditure is $e(p, \bar{u}) = \mathbf{p}\cdot\mathbf{h}(p, \bar{u}) = \min\{\bar{u}, p\sqrt{\bar{u}}\}$.

2. There are two consumers and two goods. Consumer 1 has utility $u_1(\mathbf{x}) = (x_1 x_2)^{1/2}$ and income $m_1 > 0$. Consumer 2 has utility $u_2(\mathbf{x}) = \min(x_1, 2x_2)$ and income $m_2 > 0$. Prices are $\mathbf{p} = (p_1, p_2) \gg \mathbf{0}$.

   *a*) Find the Marshallian demand functions for each consumer.

   **Answer:** Equal-weighted Cobb-Douglas utility requires consumer 1 spend half of his income on each good, so $\mathbf{x}^1(\mathbf{p}, m_1) = (m_1/2)(1/p_1, 1/p_2)$. The second consumer has Leontief preferences with $x_1 = 2x_2$ at the optimum. Total spending is then $m_2 = p_1 x_1 + p_2 x_2 = (2p_1 + p_2)x_2$ so $\mathbf{x}^2(\mathbf{p}, m_2) = (2m_2/(2p_1 + p_2), m_2/(2p_1 + p_2))$.

   *b*) Compute aggregate demand.

   **Answer:** We add the consumer demands to obating aggregate demand. It is

   $$\mathbf{x}(\mathbf{p}, m_1, m_2) = \left(\frac{m_1}{2p_1} + \frac{2m_2}{2p_1 + p_2}, \frac{m_1}{2p_2} + \frac{m_2}{2p_1 + p_2}\right).$$

   *c*) Show by example that aggregate demand cannot be written as a function of the price vector $\mathbf{p}$ and aggregate wealth $m = m_1 + m_2$.

   **Answer:** Now $\mathbf{x}(\mathbf{p}, 2, 0) = (1/p_1, 1/p_2)$ and $\mathbf{x}(\mathbf{p}, 0, 2) = (2p_1 + p_2)^{-1}(4, 2)$. Since these only agree when $2p_1 = p_2$, the different income distributions yield different demand curves.

3. Suppose $Y$ is a convex technology set (also is non-empty, closed, obeys inaction, no free lunch, and free disposal). Suppose $\mathbf{y} \gg \mathbf{0}$. Show there is a price vector with $\mathbf{p}\cdot\mathbf{y} > \pi(\mathbf{p})$ (20 points). Show that $\mathbf{p} \geq \mathbf{0}$ (5 points).

   **Answer:** Now $Y$ is a closed convex set and $\mathbf{y}$ is a point outside the set. By the Separation Theorem, there are $\mathbf{p} \neq \mathbf{0}$ and $\alpha \in \mathbb{R}$ with $\mathbf{p}\cdot\mathbf{y} > \alpha > \mathbf{p}\cdot\mathbf{z}$ for all $\mathbf{z} \in Y$. Taking the supremum over $\mathbf{z} \in Y$, we find $\mathbf{y}\cdot\mathbf{y} > \alpha \geq \pi(\mathbf{p})$.

   Now consider $-n\mathbf{e}_\ell \in Y$ by free disposal for $n > 0$. Then $\alpha > \mathbf{p}\cdot(-n\mathbf{e}_\ell) = -np_\ell$. It follows that $\alpha/n > -p_\ell$. Letting $n \to +\infty$, we find $0 \leq p_\ell$. Since $\ell$ was arbitrary, and $\mathbf{p} \neq \mathbf{0}$, $\mathbf{p} > \mathbf{0}$.

4. A consumer with $\mathfrak{X} = \mathbb{R}^2_+$ has expenditure function $e(\mathbf{p}, \bar{u}) = p_1 + p_2 + \sqrt{p_1 p_2} + (p_1 + 2p_2)\bar{u}$.

   *a*) Find the Hicksian demand function $\mathbf{h}(\mathbf{p}, \bar{u})$.

   **Answer:** Note that $e$ is concave and homogeneous of degree 1 in prices, as an expenditure function should be. We apply the Shephard-McKenzie Lemma, which says $\mathbf{h} = D_{\mathbf{p}}e$. Then

$$\mathbf{h}(\mathbf{p}, \bar{u}) = \left(1 + \frac{1}{2}\sqrt{\frac{p_2}{p_1}} + \bar{u}, 1 + \frac{1}{2}\sqrt{\frac{p_1}{p_2}} + 2\bar{u}\right)$$

   *b*) Find the indirect utility function $v(\mathbf{p}, m)$.

   **Answer:** We use the duality relation $m = e(\mathbf{p}, v(\mathbf{p}, m))$ to find indirect utility. Then $m = p_1 + p_2 + \sqrt{p_1 p_2} + (p_1 + 2p_2)v(\mathbf{p}, m)$, so

$$v(\mathbf{p}, m) = \frac{m - p_1 - p_2 - \sqrt{p_1 p_2}}{p_1 + 2p_2}$$

   *c*) Find the Marshallian demand function $\mathbf{x}(\mathbf{p}, m)$.

   **Answer:** According to Roy's Identity, $x_\ell(\mathbf{p}, m) = -(\partial v/\partial p_\ell)/(\partial v/\partial m)$. Now $\partial v/\partial m = 1/(p_1 + 2p_2)$. After a short calculation, we find

$$x_1(\mathbf{p}, m) = 1 + \frac{1}{2}\sqrt{\frac{p_2}{p_1}} + \frac{m - p_1 - p_2 - \sqrt{p_1 p_2}}{p_1 + 2p_2} = \frac{m + p_2 - \frac{1}{2}\sqrt{p_1 p_2} + p_2\sqrt{\frac{p_2}{p_1}}}{p_1 + 2p_2}$$

$$x_2(\mathbf{p}, m) = 1 + \frac{1}{2}\sqrt{\frac{p_1}{p_2}} + 2\frac{m - p_1 - p_2 - \sqrt{p_1 p_2}}{p_1 + 2p_2} = \frac{2m - p_1 - \sqrt{p_1 p_2} + \frac{p_1}{2}\sqrt{\frac{p_1}{p_2}}}{p_1 + 2p_2}$$