# Micro I Final, April 24, 2014

1. Consider the specific model with heterogeneous firms discussed in class.

   *a*) What are the gains to adding firm heterogeneity to general equilibrium model; whether it's a model of international trade or just a closed economy?

   *b*) In the trade model, what are the equilibrium conditions? Give an intuitive explanation (I'm not looking for the mathematical expressions, though you can provide them if you want).

   *c*) In the trade model, how do the two countries gain from lowering trade barriers? In what ways do they *not* gain?

2. There are two goods. Suppose a consumer has the utility function $u(x_1, x_2) = \min\{x_1, 3x_2\}$ and consumption set $\mathfrak{X} = \mathbb{R}^2_+$. Take good 1 as numéraire and let prices be $\mathbf{p} = (1, p) \gg 0$ and income be $m > 0$.

   *a*) Find the ordinary (Marshallian) demand $\mathbf{x}(\mathbf{p}, m)$.

   **Answer:** As long as $p > 0$, the optimum for these Leontief preferences requires $x_1 = 3x_2$. The budget constraint becomes $(1 + p/3)x_1 = m$, so the Marshallian demand is $x_1 = 3m/(3 + p)$ and $x_2 = m/(3 + p)$. If $p = 0$, the same formula gives the budget constraint for $x_1$, so $x_1 = m$. In that case any $x_2 \geq m/3$ is optimal.

   *b*) Compute the indirect utility function $v(\mathbf{p}, m)$.

   **Answer:** Substituting the Marshallian demand in the utility function we find $v(p, m) = 3m/(3 + p)$.

   *c*) Find the expenditure function $e(\mathbf{p}, \bar{u})$.

   **Answer:** By duality, the expenditure function solves $v(p, e(p, \bar{u})) = \bar{u}$. Thus $3e(p, \bar{u})/(3 + p) = \bar{u}$ and $e(p, \bar{u}) = (3 + p)\bar{u}/3$.

3. An exchange economy has three goods and two consumers. The consumers have preferences $u^i(\mathbf{x}^i) = (.5 - \alpha_i)\log x^i_0 + \alpha_i \log x^i_1 + .5 \log x^i_2$, where $0 < \alpha_i < .5$. Endowments are $\omega_1 = (1, 2, 1)$ and $\omega_2 = (2, 1, 1)$. Take good zero as numéraire.

   *a*) Is the economy substitutive? **Answer:** Yes. Preferences are Cobb-Douglas so demands are $\mathbf{x}^i = \mathbf{p} \cdot \omega_i((.5 - \alpha_i)/p_0, \alpha_i/p_1, .5/p_2)$. Aggregate demand is

   $$\mathbf{x} = \left( \frac{(.5 - \alpha_1)\mathbf{p} \cdot \omega_1 + (.5 - \alpha_2)\mathbf{p} \cdot \omega_2}{p_0}, \frac{\alpha_1 \mathbf{p} \cdot \omega_1 + \alpha_2 \mathbf{p} \cdot \omega_2}{p_1}, \frac{\mathbf{p} \cdot (\omega_1 + \omega_2)}{2p_2} \right).$$

   Thus $\partial x_0/\partial p_i = [(.5 - \alpha_1)\omega_{1i} + (.5 - \alpha_2)\omega_{2i}]/p_0 > 0$ for $i = 1, 2$, $\partial x_1/\partial p_i = (\alpha_1 \omega_{1i} + \alpha_2 \omega_{2i})/p_1 > 0$ for $i = 0, 2$, and $\partial x_2/\partial p_i = .5(\omega_{1i} + \omega_{2i})/p_2 > 0$ for $i = 0, 1$. This shows the economy is substitutive.

*b*) Does the economy have a unique equilibrium?

**Answer:** Yes. Cobb-Douglas preferences insure all prices are positive in equilibrium. Substitutive economies have a unique equilibrium (up to normalization of prices).

*c*) Suppose $\alpha_1$ increases. How does this affect the equilibrium prices of goods 1 and 2? For which good is the effect larger?

**Answer:** Let good 0 be the numéraire. An increase in $\alpha_1$ increases demand for good 1, while leaving demand for good 2 unchanged. Since the economy is substitutive, the prices of the both non-numéraire goods rise, with the price of good 1 rising by a larger percentage (Hicks's 2nd and 3rd Laws).

4. Suppose a firm is a price-taker, but must decide how much to produce before the market price $p$ is known. The firm knows the price distribution function $F(p)$ and cost as a function of output, $C(q)$. We presume $C$ is twice continuously differentiable with $C' > 0$ and $C'' > 0$. The firm chooses a production level that maximizes the expected utility of its owner.

*a*) Assume the firm is risk neutral. What condition must be satisfied in order to maximize expected utility.

**Answer:** If the firm is risk neutral, it maximizes expected profit. Profit is $pq - C(q)$. Expected profit is $\pi(q) = \int [pq - C(q)]\,dF(p)$. The first-order condition for profit maximization is $0 = \inf[p - C'(q)]\,dF(p) = \int p\,dF(p) - C'(q)$ or $Ep = C'(q)$. Expected price equals marginal cost. Note that $C'' > 0$ insures the second-order conditions are satisfied.

*b*) Assume the firm is risk averse. What condition must be satisfied in order to maximize expected utility.

**Answer:** In this case we maximize the expected utilty from profit. It is $\int u)pq - C(q))\,dF(p)$. The first order condition is $\int u'(p)[p - C'(q)]\,dF(p) = 0$

*c*) Comment on the differences between the two cases.

**Answer:** The presence of $u'$ in the first-order conditions makes a difference. In class we saw that when the producer is risk averse, they will produce a smaller quantity than the risk neutral producer.

5. An exchange economy has two goods and two consumers. The utility functions are $u_1(x_1, y_1) = x_1 + 2y_1$ and $u_2(x_2, y_2) = (x_2)^{1/4}(y_2)^{3/4}$. Endowments are $\omega_1 = (3, 0)$ and $\omega_2 = (0, 4)$.

*a*) Find all competitive equilibria.

**Answer:** Since the second consumer has Cobb-Douglas utilty, we know that both prices must be strictly positive in equilibrium. We normalize so that $(1, p)$ is the price vector. Consumer one will be at a corner unless $p = 2$. Consumer two has income $4p$ and demands

$\mathbf{x}_2 = (p, 3)$ Note that the market for good two will not clear if consumer one is at a corner. It follows that $(1, p) = (1, 2)$ and $\mathbf{x}_2(1, 2) = (2, 3)$. To clear the market, consumer one must demand $(3, 4) - (2, 3) = (1, 1)$. This has value 3 and is one the budget line, all of which is optimal.

*b*) Find all Pareto optima.

**Answer:** We start by considering Pareto optima in the interior of the Edgeworth box. Consumer 1 has $\mathrm{MRS}_1 = 1/2$. This must equal $\mathrm{MRS}_2 = y_2/(3x_2)$. Thus $3x_2 = 2y_2$. This line intersects the boundary of the box at the $x_1 = 2$. The segment between that intersection and 1's origin also consists of Pareto optima as the area of mutual improvement is outside the box.

The set of Pareto optima is $\{(x_1, 0) : 0 \leq x_1 \leq 2\} \cup \{(x_1, y_1) \in \mathbb{R}^2_+ : 2y_1 = -1 + 3x_1\}$.