Micro II Midterm, February 25, 2014

1. An urn contains 300 balls. 100 of them are red, the others are green or blue. Archie will draw a ball at random from the urn. Lottery A pays \$1000 if the ball is red and \$0 otherwise. Lottery B pays \$1000 if the ball is blue. Lottery C pays \$1000 if the ball is not red. Lottery D pays \$1000 if the ball is not blue. Archie's preferences are described by some increasing utility function u with $u(0) = 0$. Archie believes that drawing a blue ball has probability p. Suppose Archie prefers lottery A to lottery B. If Archie's preferences are given by an expected utility function, does Archie prefer lottery C to lottery D?

   Answer: Obviously, the probability that the ball is red is $1/3$. Note $0 \le p \le 2/3$. The expected utilities of the lotteries are $\mathsf{E}u(\mathsf{A}) = (1/3)u(1000)$, $\mathsf{E}u(\mathsf{B}) = pu(1000)$, $\mathsf{E}u(\mathsf{C}) = (2/3)u(1000)$, and $\mathsf{E}u(\mathsf{D}) = (1-p)u(1000)$. Since Archie has $\mathsf{E}u(\mathsf{A}) > \mathsf{E}u(\mathsf{B})$, we infer $p < 1/3$ (note that $u(1000) > 0$). But then, $1 - p > 2/3$, so Archie will prefer D to C.

   In practice, people tend to pick A over B and C over D, violating expected utility. This is known as the Ellsberg Paradox.

2. There are two goods. Suppose a consumer has the unusual utility function $u(x_1, x_2) = \max\{x_1, 2x_2\}$ and consumption set $\mathfrak{X} = \mathbb{R}^2_+$. Let prices be $\mathbf{p} = (1, p) \gg 0$ and income be $m > 0$.

   a) Find the ordinary (Marshallian) demand $\mathbf{x}(\mathbf{p}, m)$.

   b) Compute the indirect utility function $v(\mathbf{p}, m)$.

   c) Find the expenditure function $e(\mathbf{p}, \bar{u})$.

   Answer:

   a) Since the utility function is convex rather than concave, the optimal points will be at the corners. The possibilities are to buy only good 1, $(m, 0)$, or buy only good 2, $(0, m/p)$. The first option gives utility m while the other gives utility $2m/p$. Thus

$$\mathbf{x}\big((1,p),m\big) = \begin{cases} (m,0) & \text{if } p > 2 \\ (m,0) \text{ or } (0,m/p) & \text{if } p = 2 \\ (0,m/p) & \text{if } p < 2. \end{cases}$$

   b) The indirect utility function is then $v(\mathbf{p}, m) = m \max\{1, 2/p\}$.

   c) We use duality to find the expenditure function. $\bar{u} = v(\mathbf{p}, e(\mathbf{p}, \bar{u})) = e(\mathbf{p}, \bar{u}) \max\{1, 2/p\}$. It follows that $e(\mathbf{p}, \bar{u}) = \bar{u}/\max\{1, 2/p\} = \bar{u}\min\{1, p/2\}$.

3. There are two consumers and two goods. Consumer 1 has utility $u_1(x, y) = 3\ln x + \ln y$ and wealth $w_1 > 0$. Consumer 2 has utility $u_2(x, y) = \min\{x, y\}$ and wealth $w_2 > 0$.

a) Compute the Marshallian demand functions for each consumer.

b) Compute aggregate demand.

c) Can aggregate demand be written as a function of the price vector p and aggreate wealth $w = w_1 + w_2$?

d) Let $\alpha_i = w_i/(w_1 + w_2)$ for $i = 1, 2$. Consider the wealth distribution rule $w_i(p, w) = \alpha_i w$. Does aggregate demand obey the weak axiom of revealed preference? I.e., if $p \cdot x(p', w') \le w$ and $x(p, w) \neq x(p', w')$, can we conclude $p' \cdot x(p, w) > w'$ for all positive (p, w) and (p', w')?

Answer:

a) Consumer 1 has Cobb-Douglas utility with shares 3/4, 1/4. Thus demand is $x_1(p, w_1) = (w_1/4)(3/p_1, 1/p_2)$. Consumer 2 has Leontief prefences and so consumes equal amounts of each good. Using the budget constraint, we find $x_2(p) = (w_2/(p_1 + p_2))(1, 1)$

b) Aggregate demand is then

$$x(p) = \frac{w_1(p_1 + p_2) + 4w_2}{4(p_1 + p_2)} \left(\frac{3 + p_1}{p_1}, \frac{1 + p_2}{p_2} \right).$$

c) No. Holding aggregate wealth constant, different wealth distributions yield different aggregate demand. E.g., $w_1 = 1$ and $w_2 = 0$ and $w'_1 = 0$, $w'_2 = 1$ both have same aggregate wealth, but yield different demands (unless $p_1 + p_2 = 4$).

d) Yes. Preferences are homothetic, which implies the ULD holds for each consumer. We saw in class that this implies aggregate demand obeys WARP.

4. Suppose that production is described by a production function $f(z_1, z_2) = \sqrt{z_1} + 2\sqrt{z_2}$.

a) Let the output be good 3 and inputs goods 1 and 2. Find the corresponding production set Y.

Answer: The output obeys $y_3 \le f(z_1, z_2)$. Then the inputs are $y_1 = -z_1$ and $y_2 = -z_2$. This yields a production set of $Y = \{\mathbf{y} \in \mathbb{R}^3 : y_3 \le \sqrt{-y_1} + 2\sqrt{-y_2}, y_1 \le 0, y_2 \le 0\}$.

b) Given prices $\mathbf{p} = (p_1, p_2, p_3) \gg \mathbf{0}$, find all vectors $\mathbf{y} \in Y$ that maximize profit.

Answer: Profits are $p_1y_1 + p_2y_2 + p_3y_3$. Since $p_3 > 0$, we must choose the largest possible y_3 (for given y_1 and y_2) to maximize profits. Thus $y_3 = \sqrt{-y_1} + 2\sqrt{-y_2}$. We rewrite profits as $p_1y_1 + p_2y_2 + p_3(\sqrt{-y_1} + 2\sqrt{-y_2})$ and maximize under the constraints $y_1 \le 0$ and $y_2 \le 0$.

The Lagrangian is $\mathcal{L} = p_1y_1 + p_2y_2 + p_3(\sqrt{-y_1} + 2\sqrt{-y_2}) - \mu_1 y_1 - \mu_2 y_2$. The first-order conditions are $p_1 = p_3/2(-y_1)^{1/2} + \mu_1$ and $p_2 = p_3/(-y_2)^{1/2} + \mu_2$. Clearly, $y_1 = 0$ and $y_2 = 0$ fail to satisfy the first-order conditions. Thus $\mu_1 = \mu_2 = 0$ by complementary slackness. The first-order conditions become $p_1 = p_3/2(-y_1)^{1/2}$ and $p_2 = p_3/(-y_2)^{1/2}$.

Then $y_1 = -p_3^2/4p_1^2$, $y_2 = -p_3^2/p_2^2$ and $y_3 = p_3(1/2p_1 + 2/p_2)$.