

Micro II Final, December 10, 2015

You have until 1:45 to complete this exam. Answer all five questions. You may use any reasonable shortcuts, or any results you recall from class, the textbook, or your homework to answer the questions. To insure maximum credit, be sure to explain your answers. Each question is worth 30 points, for a total of 150 points. The questions are not equally hard. Good luck!

1. Suppose a consumer has utility $\sum_{t=0}^{\infty} \delta^t u(c_t)$ where $0 < \delta < 1$ and the felicity function is $u(c) = \ln c$. Prices are $p_0 = 2$ and $p_t = (1 + r)^{-t}$ where $r > 0$ is the interest rate. The budget constraint is $\sum_{t=0}^{\infty} p_t c_t = 123$. Let $r = 0.05$ and $\delta = 1/(1 + 0.05)$. Solve the intertemporal consumer's problem.
2. Let $\mathfrak{X} = \mathbb{R}_+^2$. The indirect utility function is $v(p, m) = (m - p_1 a_1 - p_2 a_2)/2\sqrt{p_1 p_2}$ for a_i given.
 - a) Find the expenditure function.
 - b) Find the utility function.
3. Suppose $u_1(x^1) = \max\{x_1^1, x_2^1\}$ and $u_2(x^2) = \min\{x_1^2, x_2^2\}$, with endowments $\omega^1 = (1, 1)$ and $\omega^2 = (3, 0)$. Find the core. Be careful, u_1 is not a typo!
4. Suppose an exchange economy has 2 consumers and 2 goods. Consumer one has endowment $\omega^1 = (1, 0)$. Utility is $u_1(x^1) = \sqrt{x_1^1}$. Consumer two has endowment $\omega^2 = (0, 1)$. Utility is $u_2(x^2) = \sqrt{x_1^2} + \sqrt{x_2^2}$. The consumption sets are $\mathfrak{X}_i = \mathbb{R}_+^2$. Show that there is no competitive equilibrium.
5. An economy has two goods and two identical Cobb-Douglas consumers with $u_i(x^i) = \sqrt{x_1^i x_2^i}$. The total endowment is $(0, 6)$. There is one constant returns to scale firm that produces good 1 and uses good 2 as its only input. The production function is $f(z) = 2z$.

Find all Pareto optimal allocations of goods and the corresponding net output vector.