Micro I Final, April 25, 2019

1. A firm with cost function $C(q) = q^2/2$ faces an uncertain price p. The firm chooses the production level q before the price is revealed.
   a) Write expected profit in terms of expected price Ep and the chosen quantity of output q.
   b) Suppose the firm maximizes expected profit. What quantity q maximizes expected profit?
   c) Suppose the firm maximizes the *expected utility* of profit. There is 50% chance that the price is \$10 and 50% chance that the price is \$20. Find the expected utility as a function of q, and then determine the first-order conditions for expected utility maximization in terms of the utility function u and consumption in each of the two states.

   Answer:

   a) Profit is $\pi(q) = pq - C(q) = pq - q^2/2$. Expected profit is $E\big(\pi(q)\big) = E(pq - q^2/2) = qEp - q^2/2$.
   b) Setting the derivative of expected profit to zero, we find $Ep = q$. The expected profit maximizing quantity is Ep.
   c) Under these conditions, the expected utility of profit is

   $$Eu(\pi) = \frac{1}{2}u(20q - q^2/2) + \frac{1}{2}u(10q - q^2/2).$$

   We differentiate with respect to q to find the first-order conditions.

   $$(20 - q)u'(c_1) + (10 - q)u'(c_2) = 0$$

   where $c_1 = 20q - q^2/2$ and $c_2 = 10q - q^2/2$.

2. A consumer has period utility $u(c) = \ln c$ and discount factor $\delta = 0.8$. Overall wealth is W. Suppose the budget constraint is $W \geq \sum p_t c_t$ where $p_t = (1.1)^{-t})$. Find the consumption path that maximizes utility over the budget set.

   Answer: The first-order conditions are $u'(c_t)/\delta u'(c_{t+1}) = (1.1)^{-t}/(1.1)^{-t-1} = 1.1$. Thus $c_{t+1} = .88c_t$, implying $c_t = (.88)^t c_0$. Applying the budget constraint, we obtain $W = \sum_{t=0}^{\infty}(1.1)^{-t}(.88)^t c_0 = \sum(.8)^t c_0 = c_0/(1 - 0.8) = 5c_0$. It follows that $c_0 = W/5$ and $c_t = (.88)^t W/5$.

3. A two-good, two-person production economy has utility functions $u_i(\mathbf{x}) = \ln x_1 + \ln x_2$, endowments $\boldsymbol{\omega}^1 = (3,5)$, $\boldsymbol{\omega}^2 = (2,2)$, production set $Y = \{(y_1,y_2) : y_2 \leq 0, y_1 \leq -y_2\}$, and profit shares $\theta^1 = .3$, $\theta^2 = .7$.

   a) Are there any Walrasian equilibria where nothing is produced?

   b) Find all Walrasian equilibria $(\hat{\mathbf{p}}, \hat{\mathbf{x}}^1, \hat{\mathbf{x}}^2, \hat{\mathbf{y}})$.

   Answer:

   a) We first consider whether there are any equilibria without production. Profit is $p_1y_1 + p_2y_2 \leq (p_1 - p_2)(-y_2)$. As $-y_2 \geq 0$, $\mathbf{y} = \mathbf{0}$ will only maximize profit when $p_1 \leq p_2$. Demand will be infinite if either price is zero, so we may take good one as numéraire. Then we can write $\mathbf{p} = (1,p)$. Then income is $m^1 = 3+5p$ and $m^2 = 2+2p$. Using the fact that preferences are equal-weighted Cobb-Douglas, market demand must be $\mathbf{x} = \big((5+7p)/2, (5+7p)/2p\big) \leq \boldsymbol{\omega} = (5,7)$. But then $5 + 7p \leq 10$ by market clearing for good one, so $7p \leq 5$, which contradicts $p_2 = p \geq 1 = p_1$.

   b) If the firm operates, profit maximization requires that $p_1/p_2 = 1$. We can then normalize prices with $\mathbf{p} = (1,1)$. Because production is constant returns to scale, there are no profits to consider. It follows that income is $m^1 = 8$ and $m^2 = 4$. The corresponding demands are $\mathbf{x}^1 = (4,4)$ and $\mathbf{x}^2 = (2,2)$. By market clearing, $(4,4) + (2,2) = \boldsymbol{\omega} + \mathbf{y} = (5,7) + \mathbf{y}$, so $\mathbf{y} = (+1,-1) \in Y$. It follows that the Walrasian equilibria have the form $\hat{\mathbf{p}} = \lambda(1,1)$ for some $\lambda > 0$, $\hat{\mathbf{x}}^1 = (4,4)$, $\hat{\mathbf{x}}^2 = (2,2)$, and $\hat{\mathbf{y}} = (-1,+1)$.

4. Consider a 2-good, 2-person exchange economy where each consumer has utility $u_i(\mathbf{x}) = \min\{x_1, x_2\}$. The endowments are $\boldsymbol{\omega}^1 = (3,2)$ and $\boldsymbol{\omega}^2 = (1,1)$.

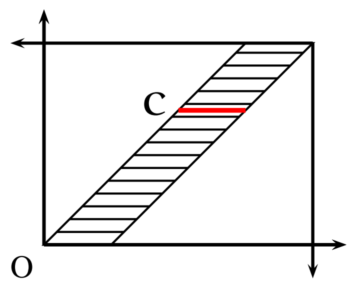
   a) Find all Pareto optimal allocations.

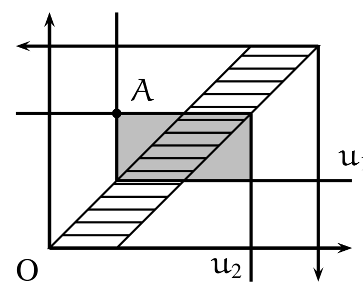
   b) Find the core.

   Answer:

   a) The aggregate endowment is $\boldsymbol{\omega} = (4,3)$. Consider the two line segments created by the corner points of the Leontief indifference curves. These have $x_1^1 = x_2^1$ and $x_1^2 = x_2^2$, as illustrated in the left diagram. Since allocations obey $x_1^2 = 4 - x_1^1$ and $x_2^2 = 3 - x_1^1$, the second line segment obeys $x_1^1 = 1 + x_2^1$. It is clear that the utility of both consumers is unchanged when we move along any horizontal line segment between these lines. Drawing the indifference curves shows that such allocations are Pareto optima.

   It is also easy to see (by drawing indifference curves) that allocations outside this diagonal region are not Pareto optima. This is illustrated on the right side of the diagram, where the gray area represents the region of Pareto improvements over an allocation at A. The set of Pareto optimal allocations is $\{(x_1^1, x_2^1), (4 - x_1^1, 3 - x_2^1) : 0 \le x_1^2 \le 3, x_1^2 \le x_1^1 \le x_1^2 + 1\}$.





b) To be in the core, an allocation must be both Pareto optimal and individually rational. The latter requires that $u_i(\mathbf{x}^i) \ge u_i(\boldsymbol{\omega}^i)$. In other words, $u_1(\mathbf{x}^1) \ge 2$ and $u_2(\mathbf{x}^2) \ge 1$. The first requires $x_2^1 \ge 2$ and the second requires $3 - x_2^1 = x_2^2 \ge 1$. In other words $x_2^1 = 2$ and $x_2^2 = 1$. The core is $\mathbf{C} = \{(x_1^1, 2), (4 - x_1^1, 1) : 2 \le x_1^1 \le 3\}$. It is the red line in the left diagram.

5. Consider a pure exchange economy with one good and two states. Endowments are $\boldsymbol{\omega}^1 = (3, 1)$ and $\boldsymbol{\omega}^2 = (0, 2)$. Both consumers have utility $u(\mathbf{x}) = \ln x_1 + \ln x_2$.

a) Find the Arrow-Debreu equilibrium $(\hat{\mathbf{p}}, \hat{\mathbf{x}}^1, \hat{\mathbf{x}}^2)$.

b) Are the consumers fully insured?

Answer:

a) With Cobb-Douglas utility, both prices must be strictly positive. We take good one as numéraire and set $\mathbf{p} = (1, p)$. Incomes are $m^1 = 3 + p$ and $m^2 = 2p$. Then demand is

$$\mathbf{x}^1 = \frac{3+p}{2}(1, 1/p) \text{ and } \mathbf{x}^2 = (p, 1).$$

Setting supply equal to demand for good one we obtain $(3 + 3p)/2 = 3$, so $p = 1$. It follows that equilibrium prices are $\hat{\mathbf{p}} = (1, 1)$. and equilibrium consumption is $\hat{\mathbf{x}}^1 = (2, 2)$ and $\mathbf{x}^2 = (1, 1)$.

b) Both consumers receive a certain consumption vector. Consumer one is certain to consume 2 units while consumer two is certain to consume 1 unit. Since consumption is independent of the state of the world, both consumers are fully insured.