Problem Set #2
ECO 3101, Fall 2013

1. Consider the following utility functions:

- $U(x, y) = xy$
- $U(x,y) = 3x + 6y$
- $U(x,y) = 20x^{1/2} + 2y$

a. Fixing y at 4, what is the marginal utility of X when x is 10, 20, and 30 for each utility function?
b. Do these utility functions exhibit diminishing marginal utility of X? Explain.
c. For each utility function, calculate the marginal rate of substitution of good X for good Y (i.e. - marginal value of X, measured in units of good Y) associated with the bundle (x, y) = (10, 4).
d. Do these utility functions exhibit diminishing marginal rates of substitution? Explain.

2. What is the "marginal rate of substitution" between two goods? Why do we expect the marginal rate of substitution to usually vary along a consumer's indifference curve?

3. Connie has a monthly income of $200 that she spends on 2 goods: meat and potatoes. Her utility function is given by: $U(m, p) = 4m + 12p$, where m stands for the quantity of meat in pounds and p stands for the quantity of potatoes in pounds.

a. If meat costs $2 per pound and potatoes cost $5 per pound, draw Connie's monthly budget line.
b. What is Connie's opportunity cost of meat, expressed in terms of potatoes?
c. How much utility would Connie get from consuming 50 pounds of meat and 20 pounds of potatoes each month? Sketch the indifference curve that passes through that bundle. *In your picture, be sure to plot at least 3 distinct points.*
d. How much utility would Connie get from consuming 80 pounds of meat and 8 pounds of potatoes each month? Sketch the indifference curve that passes through that bundle.
e. What is Connie's marginal rate of substitution of meat for potatoes (marginal value of meat, expressed in terms of potatoes)? *Hint: it is the same for every bundle*.
f. What is Connie's marginal utility of a pound of meat? Her marginal utility of a pound of potatoes?

4. What does it mean for two goods to be "perfect substitutes" for each other for a consumer? Why is the optimal bundle usually a corner solution when a consumer is choosing a bundle of perfect substitutes?

5. Jane receives utility from days spent traveling domestically (d) and days spend traveling in foreign countries (f). Her utility function is $U(d, f) = 10df$. She has $4000 per year to spend on travel. It costs $100 to spend a day traveling domestically and $400 to spend a day traveling abroad.

a. Graph Jane's budget line. *Be sure to indicate the values of both the y-axis and x-axis intercepts*.

b. What is Jane's opportunity cost of a day spent traveling in foreign countries (measured in days spent traveling domestically)?
c. Sketch the indifference curves associated with utility levels of 800 and 1200. On each curve, be sure to plot at least 3 specific bundles.
d. What is the marginal utility of a day spent traveling domestically for Jane? *Hint: your answer will be a function that depends on the number of days spent traveling in foreign countries.*
e. What is the marginal utility of a day spent traveling in a foreign country for Jane? *Here too your answer will be a function*.
f. If Jane spends 8 days traveling domestically and 8 days traveling abroad, what will be her marginal rate of substitution of foreign travel for domestic travel (i.e. – her marginal value of a day of foreign travel, measured in days of domestic travel)?
g. What is the best, affordable bundle for Jane?

6. Consider a consumer who derives utility from consuming DVD's and video games. Each month, the consumer has an amount, I, to spend on DVDs and video games. Each DVD sells for P_D. Each video game sells for P_V. Letting d denote the quantity of DVDs and v denote the quantity of video games, the consumer's utility function is $U(d, v) = 2(d^{1/2})v^{1/2}$, **which means that his marginal utility functions are: $MU_d = (v/d)^{1/2}$ and $MU_v = (d/v)^{1/2}$**.

a. Derive the consumer's demand function for DVDs (i.e - the utility-maximizing quantity of DVDs demanded, d*, as a *function* of it's exogenous determinants: I, P_D, and P_V). *Hint: with this utility function, d* won't depend on P_v.*
b. Graph the consumer's demand curve for DVD's (P_D plotted against d*) when I = \$300. Redraw the demand curve for the case when I = \$400.
c. Graph the consumer's Engel curve for DVD's (I plotted against d*) when P_D = \$20/unit. Redraw the Engel curve for the case when P_D = \$10/unit.
d. If I = \$300, P_D = \$20/unit and P_V = \$40/unit, what bundle of DVD's and video games will the consumer choose each month?
e. Suppose that initially I = \$300, and P_D = \$20/unit. If the price of DVDs falls to \$10/unit, what will happen to the quantity of DVDs demanded (up or down, and by how much)?
f. What will be the substitution effect (its magnitude and sign) of the price decline on the quantity of DVDs demanded? What will be the income effect (its magnitude and sign) of the price decline on the quantity of DVD's demanded?

7. Consider a consumer that spends her money on two goods: X and Y. Use an optimal choice diagram to illustrate the utility-maximizing response to each of the following shocks in case both X and Y are normal goods. Be sure to clearly indicate in each diagram both pre-shock and post-shock coordinates for the optimal bundle.

a. An decrease in the price of good X
b. An increase in the price of good Y
c. A decrease in income

8. What is the difference between "normal goods" and "inferior goods"? Which type of good is likely to have the larger price elasticity of demand, all else equal? Explain. *Hint: compare their income effects when price changes.*

9. Consider a consumer that chooses a bundle of leisure time (r) and a composite good (c) to maximize the utility function $U(c, r) = c + 16(r^{1/2})$, where c is the number of units of the composite good consumed, and r is the number of hours of leisure consumed each month. The consumer's monthly income equals hours of labor supplied (l^s) times the hourly wage rate (w). Every hour spent not sleeping (480 hours per month) is either consumed as leisure or supplied as labor, so the quantity of labor supplied each month is $l^s = 480 - r$. Every dollar earned from working is spent on the composite good. The wage rate (w) earned by the consumer for each hour supplied as labor is $10. The price of a unit of the composite good (p) is $20.

a. At these prices, what is the maximum amount of the composite good that the consumer could purchase each month? What is the maximum amount of leisure the consumer could consume?
b. Write down the equation for the consumer's monthly budget line, and draw it. *Be sure to indicate the magnitude of both the horizontal-axis and vertical-axis intercepts.*
c. What is the opportunity cost of an hour spent at leisure, measured in units of the composite good?
d. Under these conditions, what is the optimal bundle of leisure and the consumption good? *Hint: the marginal utility functions are $MU_c = 1$ and $MU_r = 8/(r^{1/2})$*. What is the optimal quantity of labor supplied?
e. Suppose the consumer's wage rate increases from $10/hour to $20/hour. What is the new equation for the budget line? What is the new (optimal) quantity of labor supplied?
f. What is the substitution effect (its magnitude and sign) of the wage increase on the quantity of labor supplied? What is the income effect (its magnitude and sign) of the wage increase on the quantity of labor supplied?

10. Describe the difference between the "income effect" of a price increase on quantity demanded and the "substitution effect" of a price increase on quantity demanded. Which effect will depend on whether the good is "normal"? Explain.

11. Consider the following production functions:

- $q = 3L + 2K$
- $q = 12L^{1/2}K^{1/2}$

a. Graph the short run total product curves for each production function for $0 \le L \le 25$ in the case where K is fixed at 4.
b. Graph the marginal product of labor curves for each production function for $1 \le L \le 25$.
c. Do these production functions exhibit diminishing marginal returns to labor? Explain.
d. For each production function, sketch the isoquants corresponding to production levels of 8 and 16. *Be sure to plot at least 3 points on each isoquant*.
e. For each production function, calculate the marginal rate of technical substitution of labor for capital ($MRTS_{L,K}$) at the input bundle (4,4).
f. Do these production functions exhibit diminishing marginal rates of technical substitution?

g. For each production function, determine whether it exhibits increasing returns to scale, constant returns to scale, or decreasing returns to scale.

12. What does a typical short run marginal product of labor curve look like? How about a short run total product curve? Why?

13. Use the relationships between total product, average product, and marginal product in the short run to fill out the table below.

L	**q**	**AP_L**	**MP_L**
0	0		
1	220		
2		300	270
3			
4	1100		200
5			----

14. What does it mean for a production process to exhibit "diminishing marginal returns" to all inputs? What does it mean for a production process to exhibit "decreasing returns to scale"? If a production process exhibits diminishing marginal returns to each of its inputs, will it necessarily also exhibit decreasing returns to scale? Explain.

15. Use the definitions of the various measures of cost to fill out the following table.

q	**TC**	**TVC**	**TFC**	**AC**	**MC**	**AVC**
1	18					
2					16	10
3						
4	66					
5			10	18		
6		108				