

BASIS AND THE MINIMUM VARIANCE HEDGE

Michael W. Smyser

Doctorate Student in Finance

Florida International University

(O) 305-554-2680

Robert T. Daigler

Associate Professor

Florida International University

(O) 305-554-2680

(H) 305-434-2412

BASIS AND THE MINIMUM VARIANCE HEDGEABSTRACT

Hedging the inherent price risk arising from the storage of a commodity with a futures market transaction is an effective means to control risk and therefore is an important rationale for the existence of organized futures markets. A related consideration to the theory of hedging is examining the factors affecting the simultaneous determination of spot and future prices. The purpose of this paper is to develop the basic relationships between these two topics by examining the basis formulation of the minimum variance hedge model, the seminal theory of hedging proposed by Keynes, and the equilibrium pricing dynamics obtained from the comparative statics analysis of spot and future prices developed by Stein (1961) and later extended by Bond (1984). Through the analysis of these relationships we determine the effective limitations of the minimum variance hedge (MVH) strategy under the assumptions of the price determination theory developed by Stein and Bond. In addition, the relative dynamics of the spot price, future price, and basis are examined to determine the conditions consistent with the convergence of the spot and future prices generally and under the restrictive assumption of normal backwardation.

BASIS AND THE MINIMUM VARIANCE HEDGE

Hedging the inherent price risk arising from the storage of a commodity with a futures market transaction is an effective means to control risk and therefore is an important rationale for the existence of organized futures markets. A related consideration to the theory of hedging is examining the factors affecting the simultaneous determination of spot and future prices. In this paper we draw upon the basis formulation of the standard minimum variance hedge model, Keynes (1939) seminal theory of hedging, and Stein (1961) and Bond's (1984) equilibrium pricing dynamics to develop the few basic relationships between hedging and the formalizing of spot and future prices. Through the analysis of these relationships we determine the limitations of the minimum variance hedge (MVH) strategy under the assumptions of the price determination theory developed by Stein and Bond. In addition, the relative dynamics of the spot and future prices and basis are examined to determine the conditions consistent with the convergence of the spot and future prices generally and under the restrictive assumption of normal backwardation.

The paper proceeds as follows. In section I the minimum variance hedge model originally proposed by Johnson (1960) is derived within the basis formulation. A normative framework is developed in section II which allows the identification of specific spot and future price dynamics which is consistent with a given level of correlation between the spot price change and

the change in the basis. Section III begins by examining the theory of the simultaneous determination of spot and future prices, and then analyzes some of the important comparative static results of this theory within the framework developed in section II. A summary concludes the paper.

### I. Basis Formulation of the Minimum Variance Hedge Model

The expected single period gross dollar return excluding transaction costs,  $E(U)$ , earned on the spot commodity holding  $X_S$  can be written  $E(U) = X_S \{E(S_2) - S_1\}$  where  $E(S_2)$  is the spot price expected after one period and  $S_1$  is the current spot price. The variance of the unhedged position is then simply  $\text{Var}(U) = X_S^2 \sigma_S^2$  where  $\sigma_S^2$  is the variance of the change in the spot price over time. The gross dollar return, excluding transaction costs, on a portfolio which includes both the spot position  $X_S$  and a short futures market holding  $X_F$  can be written

$$E(H) = X_S \{E(S_2) - S_1\} + X_F \{E(F_2) - F_1\} \quad (1)$$

where  $E(F_2)$  is the expected future price one period hence and  $F_1$  is the current future contract price. The variance of the portfolio gross dollar return from (1) is

$$\text{Var}(H) = X_S^2 \sigma_S^2 + X_F^2 \sigma_F^2 + 2X_S X_F \sigma_{SF} \quad (2)$$

where  $\sigma_F^2$  is the variance in the future price change and  $\sigma_{SF}$  is the covariance between the spot and future price changes. Letting  $b = -X_F/X_S$  represent the proportion of the spot position hedged,<sup>1</sup>  $\text{Var}(H)$  can be written

$$\text{Var}(H) = X_S^2 \{ \sigma_S^2 + b^2 \sigma_F^2 - 2b \sigma_{SF} \}$$

(3)

In his analysis of futures market speculation and hedging, Johnson (1960) derives the MVH ratio,  $b^*$ , by setting the partial derivative of  $\text{Var}(H)$  with respect to  $b$  to zero, with the assumption that  $X_S$  is exogenously determined or fixed, and then solving the resulting expression for  $b$ :<sup>2</sup>

$$b^* = \sigma_{SF} / \sigma_F^2$$

(4a)

$$= p_{SF} \sigma_S / \sigma_F$$

(4b)

where  $p_{SF}$  is the correlation between the spot and future price changes. From (4) we can easily determine two ranges for  $p_{SF}$  corresponding to two ranges for  $b^*$ , namely

$$0 < p_{SF} < \sigma_F / \sigma_S \equiv 0 < b^* < 1$$

(5a)

$$p_{SF} > \sigma_F / \sigma_S \equiv b^* > 1$$

(5b)

When the expected change in the future contract price is equal to the expected change in the spot price then the optimal variance minimizing strategy for the hedger is to choose  $X_F = -X_S$  or, equivalently, choose  $b=1$ . Of course, for most spot commodity and future contract markets the future price does not perfectly parallel the spot price at all times, causing an element of basis risk to directly affect the hedging decision. Following Stein (1961), Ederington (1979), and Bond (1984) we define the basis as the future price minus the spot price,  $B = F - S$ . Reformulating the Johnson (1960) MVH model into basis form, Ederington derives an expression for the gross dollar return earned on a hedged

portfolio consisting of a spot commodity holding  $X_S$  and a future contract position  $X_F$ :

$$E(H) = X_S \{ (1-b) [E(S_2) - S_1] + b [E(S_2) - S_1] - b [E(F_2) - F_1] \}$$

$$E(H) = X_S \{ (1-b) E(S) - b E(B) \} \quad (6)$$

where  $E(S) = E(S_2) - S_1$  is the expected change in the spot price and  $E(B) = E(F_2) - F_1 - [E(S_2) - S_1]$  is the expected change in the basis. Equation (6) shows that the basis formulation of the expected dollar return on a hedged portfolio is equivalent to "investing" short the amount  $b$  in the basis and simultaneously "investing" the amount  $(1-b)$  in a long spot position. As noted by Ederington (1979;p.62), when the expected change in the basis is zero then the expected return on the hedge portfolio will approach zero as  $b$  approaches one.

Using (6) to determine the variance of the dollar return for the hedge portfolio in basis form, one obtains:

$$\text{Var}(H) = X_S^2 \{ (1-b)^2 \sigma_S^2 + b^2 \sigma_B^2 - 2b(1-b) \sigma_{SB} \}$$

(7)

where  $\sigma_B^2$  is the variance of the change in the basis and  $\sigma_{SB}$  is the covariance between the change in the basis and the change in the spot price. The MVH ratio,  $b^*$ , can be obtained in a manner analogous to the derivation of (4) by taking the partial derivative of (7) with respect to  $b$  (holding  $X_S$  fixed), setting the resulting equation equal to zero and then solving for the hedge ratio  $b$ :<sup>3</sup>

$$b^* = [\sigma_S^2 + \sigma_{SB}] / [\sigma_S^2 + \sigma_B^2 + 2\sigma_{SB}]$$

(8)

An alternate derivation of (8) is provided in Appendix A.

Dividing the numerator and denominator of (8) by  $\sigma_S \sigma_B$  results in

$$b^* = [(\sigma_S/\sigma_B) + p_{SB}] / [(\sigma_S/\sigma_B) + (\sigma_B/\sigma_S) + 2p_{SB}] \quad (9)$$

where  $p_{SB}$  is the correlation between the change in the basis and the change in the spot price. Two observations are immediately apparent when (9) is examined. First,  $p_{SB} = -\sigma_B/\sigma_S$  then  $b^* = 1$ . Second, when the change in the basis is uncorrelated with the change in the spot price, i.e.  $p_{SB} = 0$ , then  $b^* < 1$ ; similarly, when  $p_{SB} > 0$  then  $0 < b^* < 1$ . Indeed, given the spot and future prices are positively correlated, one can easily obtain (see Appendix B.) the conditions on  $p_{SB}$  which completely specifies when  $b^*$  is greater than or less than one.

$$p_{SB} > 0 ; 0 < b^* < 1 \quad (10a)$$

$$-\sigma_B/\sigma_S < p_{SB} < 0 ; 0 < b^* < 1 \quad (10b)$$

$$-\sigma_S/\sigma_B < p_{SB} < -\sigma_B/\sigma_S ; b^* > 1 \quad (10c)$$

In section II we develop less obvious relationships between  $p_{SB}$ ,  $b^*$ , and the relative volatility between the spot price and the future price changes by analyzing (10a,b,c) under distinct spot and future price behavior.

## II. Properties of the Basis Formulation

In this section we develop a simple one period model to demonstrate the basic relationships between changes in the prices

of the spot and future contract to the resulting change in the basis. The model permits a normative analysis of the properties related to the temporal dynamics of the basis. The analytic framework developed in this section will be utilized in section III to relate the MVH model to the theory of a simultaneous determination of spot and future prices.

In a single period case, changes in the spot and future prices can be completely described by a given change in the basis and a given change in the spot price, or normatively by the correlation between the change in the spot price and the change in the basis,  $p_{SB}$ . For example, consider the single period spot and future contract price behavior depicted in Figure I. Both the spot and future prices are shown as decreasing over the time interval while the initial basis,  $B_1$ , is shown as negative since  $F_1 < S_1$ . The absolute change in the future contract price,  $|e_F|$ , is shown in Figure I to be less than the absolute spot price change,  $|e_S|$ ; this in turn causes the change in the basis,  $e_B = B_2 - B_1$ , to be positive, since  $B_2$  is a smaller negative number. Thus, the relative behavior of the spot price and the basis as represented in Figure I can be described normatively by the statistical property  $p_{SB} < 0$  since  $e_S < 0$  and  $e_B > 0$ . The results obtained from the above analysis are summarized in column (5) row (E) of Table I.

-----  
 Figure I and Table I about here  
 -----

The procedure outlined above, which describes the



relationship between changes in the spot and future prices and the resultant change in the basis for the single period case, is examined for the twelve distinct price dynamics characterized in Table I.<sup>4</sup> These results are based on the assumptions that the spot and future prices are positively correlated through time. Columns (2) and (3) of Table I show the results following upward trending spot and future prices, while columns (4) and (5) of the table show the results following downward trending spot and future prices. These columns are further distinguished by an initial condition that the basis is either positive or negative (contango or normal backwardation). Rows C, D, and E of the table relate to the ranges of  $p_{SB}$  given by (10a,b,c) which corresponds to  $b^*$  values greater than or less than one.

Several relationships are apparent from examining Table 1. First, as shown in row (C) of the table, when  $p_{SB} > 0$  the futures price exhibits a greater degree of volatility on average (larger absolute changes in price) than the spot price, causing the hedge ratio  $b^*$  to be less than one. Conversely, when  $p_{SB} < 0$  the spot price will exhibit a greater degree of volatility on average than the future price, however, in this case  $b^*$  can be either greater than or less than one as shown in rows (D) and (E) of table I. More specifically, when  $p_{SB} < -\sigma_B/\sigma_S$  then  $b^* > 1$  and the future price is less volatile than the spot price, which agrees with the condition on  $p_{SF}$  given by (5b) since  $p_{SF} < 1$  by definition.

The price dynamics considered above allow one to make inferences regarding the convergence of the spot and future prices. Convergence can be defined as the narrowing of the

"spread" (absolute value of the basis) and is often associated with the expiration of the future contract, since at the time of expiration the future contract price will equal the spot price.<sup>5</sup> As indicated in Table I, convergence is associated with  $p_{SB}$  and the volatility of the spot and future prices. For instance, as shown in row (C) of Table I where  $p_{SB} > 0$  and  $p_{SF} > 0$ , and the absolute change in the future price is greater on average than the absolute spot price change,  $|e_F| > |e_S|$ , the spot and future prices will converge given the initial basis is positive (negative) and the spot and future prices are on average decreasing (increasing) over time. Conversely, as shown in rows (D) and (E) of Table I where  $p_{SB} < 0$  and  $p_{SF} > 0$ , and the absolute change in the spot price is greater on average than the absolute change in the future price,  $|e_S| > |e_F|$ , the spot and future prices will converge given the initial basis is negative (positive) and the spot and future prices are on average increasing (decreasing) over time.

### III. Simultaneous Determination of Spot and Future Prices

In his classic economic study, Keynes (1936) develops a theory of hedging and proposes specific attributes which distinguish the hedger from the speculator. Keynes explicitly assumes that the hedger is motivated to enter into a futures market transaction for the purpose of reducing the price risk of holding a spot commodity. In order to induce the speculator to fulfill the opposite side of the futures market transaction, Keynes argues that the hedger must offer the future contract at a

price below that which the spot price is expected to attain at some time in the future. As the time period prior to expiration declines, the basis will increase (become less negative), since at the time of the future contract expiration the cheapest-to-deliver spot and the future price must be equal because of the delivery mechanism. Thus, the Keynesian theory predicts that the future price will be less than the spot price prior to expiration of the future contract,  $F_1 < S_1$ ; equivalently, prior to expiration the basis will be negative. Keynes terms this phenomenon the normal backwardation of spot and future prices. In this context a positive basis,  $S_1 < F_1$ , is generally referred to as contango.

The characteristics of spot and future price behavior consistent with normal backwardation can be determined by employing the framework introduced earlier in this paper and are shown in columns (3) and (5) of Table I. Given normal backwardation of spot and future prices, the conditions consistent with spot and future price convergence (narrowing of the spread) are generally limited since convergence occurs only when the change in the basis is on average positive. In particular, given normal backwardation and spot and future prices that are increasing over time, the condition consistent with convergence is for the future price changes to exhibit a relatively greater degree of volatility (larger absolute price changes) than changes in the spot price. In this case  $p_{SB} > 0$  and the MVH ratio is less than one. Conversely, when the spot and future prices are decreasing over time and normal backwardation exists, the condition that is consistent with convergence is for

the spot price changes to be more volatile (larger absolute price changes) than the future price changes and, in this case,  $p_{SB} < 0$  and the MVH ratio  $> 1$  as  $p_{SB} > -\sigma_B / \sigma_S$ .

Optimizing a firm's quadratic profit function within an econometric model, where the quantity of stocks demanded (supply of storage) is in equilibrium with the quantity of stocks in existence (demand for storage) and where the supply and demand for future contracts is also in equilibrium, Stein (1961; p.1024) employs a comparative statics analysis to "infer the nature of the forces which produce changes in spot and future prices." The comparative statics analysis of Stein's model yields several relationships concerning  $p_{SB}$ . First, when  $p_{SB} > 0$  Stein finds that the spot and future prices are expected to move together. Secondly, when  $p_{SB} < 0$  and the change in the spot price is negatively correlated with a change in the current supply of the commodity, the comparative statics analysis indicates there has been a change in the excess supply of current production. Finally, when  $p_{SB} < 0$  and the change in the spot price is positively correlated with changes in supply, Stein finds that a change in the expected spot price has occurred but there is no change in the expected price of the future contract.

Extending the model developed by Stein by incorporating rational expectations and portfolio investment behavior, Bond (1984) derives a short-term model which permits a comparative statics analysis of the effects upon the relative behavior of spot and future prices due to shocks to the rate of interest and/or to the quantity of commodity stocks held in storage. Bond

assumes that all commodity stocks held in storage are hedged (i.e. a hedge ratio of one). However, in many cases the presence of basis risk typically causes the MVH ratio to be unequal to one, i.e. the hedge ratio is not equivalent to 100% of the quantity of commodity stocks held in storage. Applying the results obtained by Bond to the normative framework developed in this paper, one is able to develop insights into the manner in which specific changes in expectations can force the MVH ratio away from one.

The analysis conducted by Bond considers temporary and permanent shocks to the rate of interest and to the quantity of stocks held in storage. Temporary shocks are defined as unanticipated, while permanent shocks are separated into unanticipated and anticipated shocks. Anticipated permanent shocks to interest rates, and temporary and both types of permanent shocks to the quantity of stocks held in storage, will cause spot and future price changes to occur in the same direction. This, in turn, relates to a positive level of correlation between the spot and future price changes and is therefore amenable to analysis under the framework developed earlier in this paper.

After further analysis Bond shows that both a temporary and an unanticipated permanent shock to the quantity of stocks held cause  $p_{SB} < 0$ ; therefore, we can relate this type of shock to rows (D) and (E) of Table I, where  $b^*$  is greater than or less than one as given by (10b,c). Moreover, anticipated permanent shocks to the quantity of stocks held or to the rate of interest cause

$p_{SB} > 0$ , which relates to row (C) of Table I where  $b^* > 1$  as given by (10a). Thus, temporary and unanticipated permanent shocks to the quantity of stocks held in storage versus anticipated permanent shocks either to the rate of interest or to the quantity of stocks held affects the MVH ratio in opposing directions by causing the MVH ratio to be, respectively, less than or greater than one. For the case of an unanticipated permanent interest rate shock, the effect upon  $p_{SB}$  is unclear, since the change in the futures price by employing the comparative statics analysis is uncertain; therefore, this type of shock could possibly produce noise within the MVH model, since the future and spot price changes could occur in opposite directions. Furthermore, while a temporary shock to the rate of interest results in  $p_{SB} > 0$ , since the change in the spot and future price are opposite in sign, the MVH model will definitely regard this type shock as noise. Thus, temporary and unanticipated permanent shock to the rate of interest can produce noise within the MVH model which can not be effectively hedged by employing the MVH model to determine the optimal proportion of stocks to hedge.

#### IV. SUMMARY

The MVH model is derived within a basis formulation. We show that given a positive MVH ratio,  $b^* > 0$ , the value of the correlation between changes in the spot price and changes in the basis,  $p_{SB}$ , completely determines whether  $b^*$  is greater than or less than one;  $b^* > 1$  as  $p_{SB} > \frac{-\sigma_B}{\sigma_S}$ . Employing a normative framework, an analysis is conducted to determine the

relationships between  $p_{SB}$  and the volatility of the spot and future prices, which results in the identification of conditions consistent with the convergence of the spot and future prices as the expiration of the future contract approaches. Employing the results of this analysis, we find that the Keynesian phenomenon of normal backwardation only occurs under a limited set of conditions that are consistent with convergence.

By analyzing the MVH model within the basis formulation, in conjunction with the theory of simultaneous determination of spot and future prices, the limitations of hedging resulting from temporary and permanent shocks to the current rate of interest and to the current quantity of a commodity held in storage may be determined. Specifically, it is shown that temporary and unanticipated permanent shocks to the quantity of stocks held versus anticipated permanent shocks either to the quantity of stocks held or to the rate of interest affects the MVH model in opposing directions by causing the MVH ratio to be, respectively, less than or greater than one. Finally, temporary interest rate shocks are regarded as noise within the minimum variance model of hedging; hence, temporary interest rate shocks can not be effectively hedged by employing the MVH for the purpose of reducing the inherent price risk arising from the storage of commodities.

FOOTNOTES

<sup>1</sup> Price risk can also occur when future consumption of the commodity is anticipated but a spot position is not currently maintained. In this case, one can form an anticipatory hedge, i.e. a futures contract for the commodity is purchased to ensure a certain price for the commodity when the future contract expires. The hedging strategy modeled in this section is the short hedge where the hedger is currently holding a spot commodity and is faced with uncertain spot prices in the future. In the context of both the anticipatory and short hedge,  $X_S$  and  $X_F$  will generally have opposite signs so that  $b$  is positive. Also, we could make the assumption here that the commodity holding  $X_S$  is one unit without affecting the result.

<sup>2</sup> Given the second derivative of  $\text{Var}(H)$  with respect to  $b$ ,  $\text{Var}(H)''$ , is greater than zero,  $b^*$  achieves a minimum. Using (3) we have:

$$\text{Var}(H)'' = 4X_S^2\sigma_F^2 > 0$$

so that  $b^*$  minimizes the function  $\text{Var}(H)$ .

<sup>3</sup> Equation (8) is identical, except for the signs on the covariance terms in both the numerator and denominator, to the asset weights solution to the well known minimum variance two asset portfolio case developed by Markowitz (1959), where the initial investment is constrained to unity. This resemblance results from the similarity between the expression for the expected return on a two asset portfolio and (6).

<sup>4</sup> The relationships shown in Table I assume that the correlation



between the changes in the spot and future prices is positive, otherwise the relationships noted will not necessarily hold.

<sup>5</sup> At expiration the future contract provides the individual that originally sold the contract an option to deliver a specific grade of commodity to an individual holding the long position. Thus, when the future contract expires a spot market transaction or the completion of the future delivery process is indistinguishable, except for transaction costs and any bias arising from a difference in liquidity between the spot and future delivery transactions. It naturally follows that in the absence of any bias due to liquidity factors or transaction costs the spot and future prices will converge upon expiration of the future contract.

REFERENCES

Gary E. Bond (1984) "The Effects of Supply and Interest Rate Shocks in Commodity Futures Markets," American Journal of Agricultural Economics, 66, pp. 294-301.

Louis H. Ederington (1979) "The Hedging Performance of the New Futures Markets," Journal of Finance, 34, pp. 157-70.

John M. Keynes (1930) Treatise on Money, Vol.II: The Applied Theory of Money, Harcourt, New York.

Leland L. Johnson (1960) "The Theory of Hedging and Speculation in Commodity Futures," Review of Economic Studies, 27, pp. 139-51.

Harry Markowitz (1959) Portfolio Selection: Efficient Diversification of Investment, John Wiley and Sons, New York.

Jerome L. Stein (1961) "The Simultaneous Determination of Spot and Futures Prices," American Economic Review, 51, pp. 1012-25.

APPENDICES

Appendix A. This Appendix derives equation (8).

The expected change in the basis is defined in the text as  $E[B] = E[B_2 - B_1] = E[F_2 - S_2] - (F_1 - S_1) = E[F_2 - F_1] - E[S_2 - S_1]$ . Therefore the variance in the change in the basis can be expressed as:

$$\sigma_B^2 = \sigma_F^2 + \sigma_S^2 - 2\sigma_{SF} \quad (\text{A-1})$$

1)

Since  $E[(S_2 - S_1)(B_2 - B_1)] = E[(S_2 - S_1)(F_2 - F_1) - (S_2 - S_1)(S_2 - S_1)]$ , we can express the covariance between changes in the spot price and changes in the basis as

$$\sigma_{SB} = \sigma_{SF} - \sigma_S^2 \quad (\text{A-2})$$

2)

Substituting (A-1) and (A-2) into (4a) results in (8).

Appendix B. This Appendix determines the conditions (10a & b).

Given  $b^* > 0$  it follows from (4a) that  $\sigma_{SF} > 0$ . Therefore, using (A-2), we can write:

$$b^* > 0 = \sigma_{SF} = \sigma_S^2 + \sigma_{SB} > 0 \quad (\text{B-1})$$

1)

$$\text{or} \quad b^* > 0 = p_{SB} + \sigma_S / \sigma_B > 0 \quad (\text{B-2})$$

2)

Substituting (A-2) in (A-1) we have  $\sigma_B^2 = \sigma_F^2 - \sigma_S^2 - 2\sigma_{SB}$  [or  $\sigma_B^2 = \sigma_F^2 + \sigma_S^2 - 2(\sigma_{SB} - \sigma_S^2)$ ]. Therefore,

$$\sigma_F^2 = \sigma_S^2 + \sigma_B^2 + 2\sigma_{SB} > 0 \quad (\text{B-3})$$

3)

$$\text{or} \quad \sigma_S / \sigma_B + \sigma_B / \sigma_S + 2p_{SB} > 0 \quad (\text{B-4})$$

4)

Using (9) to express  $b^*$  and given (B-2), and (B-4) we can write

$$0 < b^* < 1 \equiv \sigma_S/\sigma_B + p_{SB} < \sigma_S/\sigma_B + \sigma_B/\sigma_S + 2p_{SB} \quad (\text{B-5})$$

5)

and 
$$b^* > 1 \equiv \sigma_S/\sigma_B + p_{SB} > \sigma_S/\sigma_B + \sigma_B/\sigma_S + 2p_{SB} \quad (\text{B-6})$$

6)

Rearranging (B-5), results in (10a). Given (B-2) and rearranging (B-6) results in (10b).

## ELIMINATED FROM TXT

Johnson (1960) derives a measure of hedging effectiveness,  $e$ ,  $0 \leq e \leq 1$ , by determining the percentage reduction in the variance of the unhedged spot position resulting from a hedging strategy employing the MVH ratio,  $b^*$ , as the proportion of the spot position hedged by an offsetting future contract position.

$$\begin{aligned} e &= 1 - [\text{Var}(R) / \text{Var}(U)] \\ &= p_{SF}^2 \end{aligned}$$

(11)

Expression (11) shows that the effectiveness of the MVH improves directly with an increase in  $p_{SF}$ , consequently the MVH model associates nonlinear spot and future price behavior as noise. This observation can be attributed to an alternate solution for  $b^*$  given by the slope coefficient found by least squares regression of the spot price changes onto the future price changes. The sample coefficient of determination obtained from such a least squares regression is, of course, equivalent to (11) and is a measure of the linear association between the spot and future price changes. It will be shown in section III of this paper that under certain assumptions specific factors creating noise within the MVH model can be identified. However, before considering this issue it is necessary to first develop a suitable analytic framework.

???? difficult to determine partial derivative of  $e$  to covariance of spot and basis since all variables are

related. ????

Expression (11) represents the basis formulation of the Johnson measure of hedging effectiveness.

$$e = [\sigma_S^2 + \sigma_{SB}]^2 / [\sigma_S^2 + \sigma_B^2 + 2\sigma_{SB}] \sigma_S^2$$

(11)

$$= b^* \{1 + \rho_{SB} \sigma_B / \sigma_S\}$$

(12)

When  $b^* > 1$  in (12) then  $\rho_{SB} < 0$ , otherwise  $e > 1$  which can not result, so that the condition expressed in (9c) is consistent with  $e < 1$ .

In their empirical study of basis risk in the Canadian feeder cattle trade, Carter and Loyns (1985) discuss the relationship between hedging effectiveness and the volatility in the basis:

"The coefficient of determination measures the proportion of the

variance in cash price changes that the futures price changes

explain, and thus is positively related to the stability of the

basis." (Carter and Loyns, p.38)

A more detailed analysis of (11) than presented here might provide direct theoretical support for the intuitively appealing statement of Carter et.al.

C.A. Carter and R.M.A. Loyns (1985) "Hedging Feedlot Cattle: A

Canadian Perspective," American Journal of Agricultural Economics, pp.32-9.

$\infty \cdot \langle \rangle \beta > \mu \Phi \approx \{ \theta \Gamma \Sigma \delta \sigma \alpha^2 \Omega \epsilon \div \tau \geq \leq \sqrt{\Pi} \}$  201-A

$\bullet \cdot x \neq \textcircled{\leq} > \langle \ell \approx \downarrow \rightarrow \leftarrow \uparrow \nabla \textcircled{\geq} \geq^1 \geq^2 \geq^3 \}$  201-B

$\zeta \xi \chi \} \beta \nu \mu \lambda \kappa \{ \theta \gamma \phi \delta \sigma \alpha \psi \omega \epsilon \rho \tau \eta \iota \omicron \pi \}$  201-C

$\infty \Xi \neq \} \leq > \langle \geq \Sigma \Delta \Phi \Gamma \Theta \{ \infty \Lambda \Psi \Omega \equiv \div \bullet \Upsilon \partial \int \sqrt{\Pi} \}$  201-D

$\epsilon \tau \geq \leq \sqrt{\Pi} \alpha \sigma \delta \Sigma \Gamma \Theta \{ \approx \Phi \infty \cdot \langle \rangle \beta > \mu \}$  203-A

$\bullet \cdot x \neq \textcircled{\leq} > \langle \ell \approx \downarrow \rightarrow \leftarrow \uparrow \nabla \textcircled{\geq} \geq^3 \}$  203-B

$\zeta \xi \chi \} \beta \nu \mu \lambda \kappa \{ \theta \gamma \phi \delta \sigma \alpha \epsilon \tau \eta \iota \omicron \pi \}$  203-C

$\infty \Xi \neq \} \leq > \langle \Lambda \infty \{ \Theta \Gamma \Phi \Delta \Sigma \geq \equiv \bullet \Upsilon \partial \int \sqrt{\Pi} \}$  203-D

alt-D w/ 203-D

e alt-E w/ 201-C

<sup>2</sup> alt-Q w/ 201-A

$\epsilon_F$