A BIVARIATE GARCH APPROACH TO THE FUTURES VOLUME-VOLATILITY ISSUE

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ABSTRACT

This paper examines the volume-volatility-type of trader relationship by employing daily volume of five futures contracts segregated into four types of traders. This breakdown of total volume into its components allows us to test whether one or more groups can be simultaneously associated with the level of volatility on the conditional return. A bivariate GARCH model is employed to examine the relationships between the category volumes and to account for clustering of volatility. Overall, while both volume and volatility have ARCH effects, their relative importance in the return relationship depends on the type of futures contract and category of trader.

I. Introduction

The relationship between information, volatility, volume and return has received considerable attention in the literature. The purpose of this paper is to investigate whether the total risk in futures prices can be modeled as time-varying, and whether this time variation can be separately allocated to the volume and volatility components of the market.

The importance of this relationship is directly related to the role of information in price formation, with volatility and volume providing measures of the significance of the information reflected in the market. Bookstaber and Pomerantz [1989], Grossman and Stiglitz [1980], Huffman [1992], Jang and Ro [1989], Richardson and Smith [1994], Ross [1989], and Wang [1994] all develop models which show the importance of volume and/or the volatility-volume relationship in reflecting changes in the information/beliefs of traders in the marketplace.[1] In addition, previous empirical studies on stock market volatility include attempts to separate the importance of private

[1] Related sets of models in the information literature are rational expectations models and asymmetric information models. Rational expectation models associate prices to private information signals, while asymmetric information models emphasize intraday relationships. Rational expectations models typically treat volume as a byproduct of the market mechanism. The intraday asymmetric models show that volume will concentrate at certain times within the day, creating the familiar U-shaped volume and volatility curves. See Grossman [1989] for a collection of papers examining rational expectations models. Admati and Pfleiderer [1988] and Kyle [1985]) are examples of intraday asymmetric models. See Admati [1991] for a review of both types of models.

information, public information, and noise (for example, see French and Roll [1986]).

The roles of information and noise (and hence the resultant volume-volatility relationship) are less straightforward for futures markets. Typically private information for futures markets is associated with superior analysis rather than factual private knowledge. Public announcements (information) result in changes in positions due to differing beliefs concerning the importance of that information. Scalpers (floor traders) provide liquidity to the markets and hence respond to an increase in trades by other types of traders by rebalancing their inventory position. Hedgers use futures as a means to rebalance their cash portfolios in order to reduce risk via a short hedge or to lock-in the future price of purchase via a long hedge. Hedgers' demand is based on a combination of changes in beliefs and liquidity requirements. The general public (speculators) try to profit based on their beliefs of future price movements, which often are interpreted from charts of past prices or other technical trading methods. Speculators response to price movements assumes that the price changes are due to "information" rather than liquidity demand. Therefore, information for futures markets is related to the effect of public announcements and/or changes in beliefs rather than the traditional definition of private information. Moreover, noise can be closely associated with the widespread use of technical analysis and uninformed traders. Such noise is consistent with the behavior of small speculators (the general public). Consequently, the models of speculative and hedging behavior (differences of opinion/beliefs), such as those by Harris and Raviv [1993] and Shalen [1993], are more appropriate for futures markets than the rational expectations models which incorporate private information.

This paper extends the volatility-volume literature by employing a unique data set that separates volume into four types of trades/traders: scalpers, commercial traders, other floor trades, and the general public. This breakdown into type of trader allows us to identify which group(s) of traders is (are) closely associated with changes in volatility. In addition, segregation of the volume into groups provides a more precise examination of the volume-volatility statistical association. Thus, this data allows us to test Bessembinder and Seguin's [1993] argument that the volatility-volume relationship may depend on the type of trader.

This volume and volatility data are examined in a bivariate GARCH (Generalized Autoregressive Conditonal Heteroskedasticity) framework, as discussed by Engle et.al. [1984] and Bollerslev et. al. [1988], in order to examine the interrelated characteristics of these two series. Employing a bivariate GARCH model provides insights into the interactions that are not apparent in an ordinary least squares model. Specifically, this approach provides estimates of the importance of volume and volatility conditional upon past volatility information of each of these variables. We follow the approach of Giannopoulous (1995) in examining the interrelationships of two variables likely to affect the conditional mean return.

Empirical aspects of the volume-volatility relationship in the literature for various instruments have been examined by Bessembinder and Seguin [1993], Chang and Schachter [1992], Gallant et al. [1992], Harris and Raviv [1993], Jain and Joh [1988], Karpoff [1987], Lang et al. [1992], and Schwert [1989]. These studies consistently show that a significant relationship exists between volume and volatility, with volatility measured as the absolute price change or the squared price change. We extend their findings by examining the interactions between volume and volatility when volume is separated by type of trader using bivariate GARCH to model the interactions of the two variables in order to decompose their effects.

II. The Volatility-Volume Relationship

A. Models

Three interrelated groups of theories exist to explain the volatility-volume relationship. The first is loosely called information theories, since information is the driving force that determines both volume and volatility. The Mixture of Distribution and the Sequential Arrival of Information theories are information theories. The second category is labeled trading theories and includes the Admati and Pfleiderer [1988] versus Brock and Kleidon [1992] debate as to the cause of the volatility-volume relationship.[2] The third category is the argument for the dispersion of beliefs/expectations, as explained by Harris and Raviv [1993] and Shalen [1993].

The Mixture of Distributions model, associated with Clark [1973], Epps and Epps [1976], Tauchen and Pitts [1983], and Harris [1986], is based on the assumption that the variance per transaction is monotonically related to the volume of that transaction. Further, it assumes that a mixing variable is the cause of the joint volatility-volume relationship. Often the number (and implicitly the importance) of information arrivals are designated as the mixing variable, although the volume per transaction and the number of transactions also have been designated as mixing variables.

Each model contains unique features. Clark's model employs volume as a proxy for the speed of information flow. He associates volume and volatility on a contemporaneous basis, with no causal relationship between them. Clark's model implies that all groups who trade on information will have a similar relationship between volume and volatility. Epps and Epps' model is based on the disagreement between traders: the greater the disagreement, the larger the level of trading volume. Epps and Epps suggest a causal relationship from volume to volatility. Also, their model implies that groups with greater disagreement will have a more pronounced relationship between volatility and volume.

The Mixture of Distributions model has received the most attention in the literature for the volatility-volume studies. Harris [1987] and Tauchen and Pitts [1983] show that the joint distribution

[2] As noted above, the Admati and Pfleiderer model is associated with asymmetric information. However, since trader behavior is the key element of their model we categorize it as a trader model rather than an information model.

of changes in price (variability) and volume are modeled as a mixture of bivariate normal distributions and show why the variance or absolute price change is a function of volume. However, other variables, such as the number of transactions (Harris [1987]), are also suggested as mixing variables.

The Sequential Arrival of Information model is developed and extended by Copeland [1976, 1977], Jennings and Barry [1983], Jennings, Starks, and Fellingham [1981], and Morse [1981]. This model assumes that information is disseminated sequentially from one group to another. This movement of information creates numerous price changes while also creating volume. It also implies the continuation of higher volatility after the initial information shock rather than spikes in volatility.

Admati and Pfleiderer [1988] and Kyle [1985] provide trading behavior models by associating the timing of informed trades with the size of uninformed volume. Consequently, Admati and Pfleiderer show that trading is bunched in time, which justifies the intraday U-shape volume and volatility curves prevalent in the literature. Brock and Kleidon [1992] associate the U-shape curves to market closure, the power of dealers, and portfolio rebalancings.

Harris and Raviv [1993] and Shalen [1993] develop the dispersion of beliefs/expectations as the key factor determining the additional volatility and additional expected volume associated with noisy information (as well as developing other trading behavior relationships in the futures market). These concepts are developed more fully below.

B. Dispersion of Beliefs

Heterogeneous models of trader behavior typically involve differing beliefs concerning the importance of information and when traders act on public information. Public information per se is impounded quickly within futures prices (typically within 30 minutes; see Ederington and Lee

[1993]). Therefore, the key factor for futures markets is how differing <u>beliefs</u> are impounded into prices and how those beliefs are used to form expectations.

We assume that private knowledge of public macroeconomic announcements is not available for futures markets participants (as is supported by Daigler [1994]). A greater dispersion of beliefs determines the excess variability of prices and the excess volume of trade, as influenced by the type of trader group in question, as modeled by Harris and Raviv [1993] and Shalen [1993]. The dispersion of beliefs is caused both by fundamental traders and speculative traders. But the underlying causes of the dispersion of beliefs differ for these two groups.

"Informed" traders are those who have the resources to evaluate fundamental information and its impact on future prices. These traders have a relatively homogeneous set of beliefs and therefore trade within a relatively small range of prices. In addition, their revision of beliefs, given a shock due to new public information, causes prices to damp out in a relatively short period of time. We associate informed traders with commercial traders, who have the resources to gather data and undertake sophisticated fundamental analysis.

Uninformed traders are those who use more ad hoc methods of "analysis," such as technical analysis. In fact, the use of technical analysis is widespread in futures markets, including commodity funds that employ technical trading models as their main "tool" for trading decisions. The result of this ad-hoc analysis is a wider dispersion of beliefs, resulting in a greater variability of prices. In addition, technical traders react to all changes in prices as if they reflect informative trading, while the demand of liquidity traders (such as certain types of hedgers) create noise in price movements. Thus, technical (uninformed) traders exaggerate that noise.

The revision of beliefs of the uninformed traders cause prices to damp out more slowly than those of the informed investors. This slower damping process is due precisely to the fact that technical analysis signals are constantly in revision as new "information" (i.e. price movements) occur. In addition, uniformed traders react to a shock in prices (and therefore the need to revise beliefs) at different times. This process of trading is consistent with French and Roll [1986], who state that traders overreact to each others trades, as well as Grundy and McNichols [1989] and Shalen [1993] who show that speculators' revision of beliefs cause trading in more than one period. We associate uninformed traders with the general public, who are known to use technical analysis for most of their trading decisions. The interaction between volatility and price reflects the impounding of information, and should thus differ by trader type.

This description of informed and uninformed traders is similar to Brock and LeBaron's [1993], Harris and Raviv's [1993], and Shalen's [1993] descriptions of how beliefs affect trading behavior. In particular, Shalen's model relates excess volatility both to speculators' divergent beliefs and to their trading on the noisy liquidity demand of hedgers. Moreover, Brock [1993] develops a theoretical noise trading model where volatility bursts are related to volume across differing groups. Our data should contain important information if these models are correct.

C. Conditional Variance Effects

Autoregressive conditional heteroskedasticity (ARCH) has been identified as a common occurrence in financial market data. The persistence of variance after price shocks has become an important issue in the examination of the volume-volatility studies. An explanation for the presence of ARCH effects is that daily returns are generated by a mixture of distributions, where the rate of daily information arrival is the stochastic mixing variable. Diebold [1986], Gallant, Hsieh, and Tauchen [1988], Lamoureux and Lastrapes [1990] suggest that the ARCH effects might capture the entire time series properties of the information mixing variable. In particular, Lamoureux and Lastrapes [1990] show that ARCH follows from the serial correlation of the mixing variable - the number of price changes (which represents the number of information arrivals). However, Gannon

[1994] and Lamoureux and Lastrapes [1990] find that the ARCH effects disappear when volume is also employed as a proxy for information. Conversely, Bessembinder and Seguin [1992, 1993] and Lamoureux and Lastrapes [1994] show that adding volume as a variable is not sufficient to remove the ARCH effects in variance. Consequently, whether volume adequately explains the information found in volatility, or whether conditional variance effects can be partly or fully explained by an ARCH model is not completely settled. We propose two modifications that should better resolve this argument. First, accounting for GARCH effects when volume is segregated by type of trader will allow the model to more precisely describe the relationship. Second, estimating GARCH models simultaneously for volume and volatility will allow the consideration of covariance between the two variables.

III. Data and Volatility Measure

Two types of data are employed in this study. First, time and sales prices for futures provide the relevant information to determine the volatility measures. Second, the volume data separates traded futures volume into four types of traders:

1. CTI1: volume for the local floor trader's own account or for an account which (s)he controls, i.e. scalpers and other floor traders.
2. CTI2: volume for the clearing member's house account, i.e. commercial clearing members.
3. CTI3: volume for the broker executing trades for other brokers present on the exchange floor, or an account controlled by other such brokers, i.e. members filling orders for other members.
4. CTI4: volume for any other type of customer; i.e. members filling orders for the public.

The volume data is examined using data spanning two years for five financial futures contracts, where the contracts are silver, the MMI stock index futures, Muni bonds, T-notes, and T-bond futures contracts.

The futures contract expiration month with the highest open interest is employed in the analysis in order to concentrate on the most active contract month.[3] The interest rate contracts are rolled over near the first of the expiration month. The MMI contract is rolled over at expiration. The largest open interest for silver does not follow a nearby pattern, since the December contract often had the largest open interest from summer until the end of November.

The measure of volatility employed is the Garman-Klass [1980] volatility measure. The Garman and Klass measure of volatility is eight times more efficient than using the last price in each time interval to obtain a measure of volatility:

$$\text{Volatil} = \text{Var(GK)} = \tfrac{1}{2}\,[\ln(\text{High}) - \ln(\text{Low})]^2 - [2\ln(2) - 1]\,[\ln(\text{Open}) - \ln(\text{Close})]^2 \qquad (1)$$

Where:

Var(GK) = the variance using the Garman-Klass method

ln = the natural logarithm

High, Low, Open, Close = the open, high, low, and closing prices in the interval being used to determine the volatility.

IV. The Bivariate GARCH Model

A bivariate GARCH model is chosen in order to examine the potential interrelationships among the four types of traders in the volume-volatility relationship. Both volatility and volume

[3] Using the expiration month with largest volume as the selection criteria would create the unwanted feature of often skipping from one expiration month to another for the less liquid futures contracts. Moreover, using the largest open interest month avoids using a contract during its expiration period. Alternatively, lumping all expirations together would create difficulties in accurately measuring a combined price change, as well as possibly obscuring the true relationship. In any case, the expiration month with the largest open interest typically dominates trading activity (volume).

exhibit ARCH effects; i.e. there is a clustering of the volatility of the return and the volatility of the volume series over time, where the clustering of these series should be related if volume and return volatility are related. We assume that returns to futures trading can be expressed as a function of the set of variables conveying information to the market multiplied by the set of coefficients plus an error term. The errors have an expected value of zero conditional on prior period information, and the conditional variance of the error term is a function of past period squared errors. The interrelationships and clustering factors dictate that the bivariate GARCH model be determined as follows:

$$VOLATIL_{,t} = \beta_{VOLATIL}\, VOLATIL_{t-1} + \epsilon_{VOLATIL,\,t} \tag{2}$$

$$CTIX_{,t} = \beta_{CTIX}\, CTIX_{,t-1} + \epsilon_{CTIX,\,t} \tag{3}$$

The vector parameterization of the error terms is

$$\epsilon_t|(\epsilon_{t-1},..) \sim N(0,\mathbf{H}_t)$$

where $\mathbf{H}_t$, the conditional variance-covariance matrix, is positive definite with

$$h_{11,t} = \Omega_{11} + \alpha_{11}\, \epsilon^2_{\ VOLATIL,t-1} + \beta_{11}\, h_{11,t-1}$$

$$h_{22,t} = \Omega_{22} + \alpha_{22}\, \epsilon^2_{\ CTIX,t-1} + \beta_{22}\, h_{22,t-1}$$

$$h_{12,t} = \Omega_{12} + \alpha_{12}\, \epsilon^2_{VOLATIL,t-1}\, \epsilon^2_{CTIX,t-1} + \beta_{12}\, h_{12,t-1} \tag{4}$$

In this model:

$VOLATIL_t$ = the level of Garman-Klass volatility at time t

$CTIX_t$ = the level of volume for CTI category X at time t

ϵ_t = the error term for CTIX or VOLATIL at time t

h = the variance effects of CTIX or VOLATIL

Equations (2) through (4) are solved simultaneously.

In this formulation $h_{11,t}$ represents the conditional variance of the Garman-Klass volatility measure and $h_{22,t}$ is the conditional variance of volume for a specific trader category. Then $h_{12,t}$ is the covariance between volatility and volume. Following Giannopoulous, $h_{12,t}/h_{11,t}$ can represent the proportion of conditional volatility attributable to the CTI category volume. Phrased alternatively, this is the sensitivity of conditional volatility to category volume.

This model captures the individual effects of volume and volatility, as well as their interaction effects, on any serial dependence in the residuals of the OLS equations. Significant coefficients indicate which terms add information to the forecast at time *t* of volume and/or volatility. Thus, the bivariate GARCH formulation considers current variance, past variance, and the past error of the forecast of the variable in question (as well as information on the variance and error of the other variable and the covariability effects) to adjust the current forecast of the variable in question. In order to estimate this model, we assume that, for any group of traders, volume and volatility will simultaneously affect a conditional mean that is held constant for estimation purposes, and that volume and volatility are the primary variables through which information is conveyed in futures markets.

IV. Empirical Results for the Volatility-Volume Relationship

A. Basic Volatility-Volume Results and Descriptive Statistics

Table 1 provides descriptive statistics concerning percentage breakdown of the volume into the four CTI categories for each of the five futures contracts. The relative importance of each CTI category and the differing proportions across contracts provide interesting results. Categories I (scalpers) and IV (general public) trade the most in all of the contracts examined, while category III (orders for other traders) shows the smallest levels of trading volume. Surprisingly, commercials (CTI2) have a relatively smaller percentage importance for T-bonds as compared to the other futures contracts. On the other hand, commercials are relatively important for the MMI, Muni, and

T-note contracts. Also, silver futures contract has a lower CTI2 proportion than CTI3 proportion (which is the reverse of the more liquid contracts), and silver has the highest proportions of all futures contracts for the CTI3 and CTI4 categories. The MMI, T-bond, and T-note futures all have a large proportion of commercial clearing member activity (CTI2) volume.

[SEE TABLE 1]

Table 2 examines univariate statistics on the volume and volatility series for all five contracts. All volume and volatility values are significantly non-normal, as shown by Jarque-Bera tests, indicating that OLS models to explore relationships are likely to be misspecified. We hypothesize that the cause of non-normality is likely to be autoregressive conditional heteroskedasticity.

[SEE TABLE 2]

We further examine each series by testing for ARCH processes. Engle proposes a test for ARCH based on the presence of serial correlation in the residuals of a series. This test is performed by regressing the series on a constant and then regressing the squared residuals from the first regression on a constant and own lags.

Table 3 presents the results of these tests for up to 5 lags, so that day of the week effects are fully captured. As the table shows, ARCH effects are present for at least the first lag in all series. Some series have multiple significant lags. This indicates there is important information that models not accounting for ARCH may fail to model properly.

[SEE TABLE 3]

B. *Bivariate GARCH Results*

Tables 4 through 8 provide the parameter values for the bivariate GARCH results to examine the autoregressive conditional heteroskedasticity, and interaction effects that exist for the volume and volatility data for each of the five different futures contracts studied. Most equations show significant values for α and β, the ARCH and GARCH coefficients respectively, verifying the autoregressive conditional heteroskedasticities in these series.[4] Overall, while both volume and volatility have ARCH effects, their relative importance depends on the type of futures contract and category of trader. In fact, no consistent pattern emerges in terms of which coefficients are significant across different futures contracts.

It is also notable that the cross terms in the h_{12} equation are significant for many of the series. Thus, the bivariate model contains more information as well as identifying the portion of conditional return attributable to trader category volume.

[SEE TABLES 4 TO 8]

V. Conclusions and Implications

This study complements the results of earlier studies on the volume/volatility relationship in financial markets. Not only are volume and volatility interrelated in the transmission of information within markets, but also they follow a bivariate GARCH process. This process is even more apparent when volume is broken into trader components. These GARCH effects are important information that should be considered when modeling the conditional return series.

Future work should consider the incorporation of the type-of-trader volume into the return generating process as well as the revision of beliefs that occurs following shocks to volume,

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[4] A GARCH (1,1) model is fit: thus, only a one period lag is employed in the estimation. Using one lag is consistent with the findings in the literature and the results shown in Table 3, and makes the results tractable for presentation. The smaller beta coefficients that exist for most of the volatility series as compared to the associated volume series can be related to the greater proportional standard deviations of volatility. Similarly, T-bonds have a smaller proportional standard deviation of volatility - and also much larger beta coefficients for volatility. However, care must be taken in interpreting all of the coefficients, since equations (2) to (4) are solved simultaneously. For example, series with larger GARCH effects for volatility will have smaller β_{CTIX} effects.

volatility or both.

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Table 1
Average Trader Category Volume as a Percentage of Total Volume

Series	CTI1	CTI2	CTI3	CTI4	Total Avg Volume
Silver	45.98%	6.56%	10.70%	36.76%	1,854
MMI	50.37%	19.56%	7.18%	22.89%	13,528
Munis	50.43%	18.60%	1.40%	29.57%	9,364
T-Notes	45.93%	20.31%	6.98%	26.79%	31,260
T-Bonds	55.49%	13.35%	7.11%	24.05%	442,991

Table 2
Univariate Statistics on
Category Volume and Garman-Klass Volatility for Five Futures Contracts

Panel A: Silver

Mean	Median	Maximum	Minimum	Std. Dev	Skewness	Kurtosis	Jarque-Bera	P-Value
873.1366	662.0000	4716.000	32.00000	704.2441	1.499314	5.787656	337.3513	0.00000
125.3064	22.00000	1499.000	0.000000	229.1642	2.972552	13.09233	2761.138	0.00000
205.3271	66.00000	3253.000	0.000000	355.6344	3.384071	19.28896	6261.651	0.00000
697.4017	509.0000	4145.000	30.00000	608.9392	1.790589	7.403125	648.2736	0.00000
2.719264	1.596278	24.15846	0.000000	3.200903	2.842838	13.02365	2672.611	0.00000

Panel B: MMI

Mean	Median	Maximum	Minimum	Std. Dev	Skewness	Kurtosis	Jarque-Bera	P-Value
6931.345	6263.000	24078.00	670.0000	3640.769	1.096051	4.675727	157.0259	0.00000
2685.584	2505.000	8732.000	356.0000	1425.520	1.060988	4.424505	134.7225	0.00000
985.0667	886.0000	2809.000	107.0000	529.6749	0.884745	3.431812	68.42465	0.00000
3136.982	2405.000	12845.00	376.0000	2056.709	1.358125	4.778636	217.4198	0.00000
3.677301	0.794766	515.6380	0.085196	27.36709	15.43003	265.4978	1440809.	0.00000

Panel C: Munis

Mean	Median	Maximum	Minimum	Std. Dev	Skewness	Kurtosis	Jarque-Bera	P-Value
4812.398	4186.000	14768.00	243.0000	2608.025	1.042465	3.932189	107.5782	0.00000
1773.174	1456.000	7017.000	104.0000	1152.404	1.693115	6.314315	463.0567	0.00000
132.7232	92.00000	850.0000	0.000000	139.3107	2.110678	8.327109	952.8325	0.00000
2813.002	2423.000	12582.00	323.0000	1672.738	1.946745	8.735758	991.1999	0.00000
0.370145	0.213687	5.330997	0.013996	0.564341	4.738979	31.53366	18645.03	0.00000

Panel D: Treasury Notes

Mean	Median	Maximum	Minimum	Std. Dev	Skewness	Kurtosis	Jarque-Bera	P-Value
14749.42	13399.50	53151.00	3360.000	6798.504	1.754092	8.201866	803.7381	0.00000
6518.800	6019.500	25279.00	1169.000	3063.163	1.776009	9.387224	1090.525	0.00000
2229.335	1922.000	11417.00	242.0000	1453.930	2.355444	11.84608	2050.763	0.00000
8577.267	7624.000	34243.00	2087.000	4157.751	1.836137	8.815318	965.7801	0.00000
0.248912	0.128471	4.545449	0.005658	0.427462	5.662067	44.25438	37365.76	0.00000

Panel E: Treasury Bonds

Mean	Median	Maximum	Minimum	Std. Dev	Skewness	Kurtosis	Jarque-Bera	P-Value
256971.2	240486.0	650250.0	89883.00	89775.70	0.912763	4.057495	88.08953	0.00000
61687.60	57126.00	165041.0	20418.00	23036.12	1.047113	4.308861	120.7074	0.00000
32725.72	30602.00	95513.00	11545.00	12310.09	1.313959	5.495614	259.9446	0.00000
111071.7	102542.0	298104.0	41712.00	41186.16	1.061031	4.389726	127.3491	0.00000
0.582084	0.365677	5.862516	0.032182	0.670614	3.414020	18.84264	5890.224	0.00000

Univariate descriptive statistics on daily observations for five futures contracts.

Table 3
Test for ARCH Processes

Panel A - Silver

	C	Lag 1	Lag 2	Lag 3	Lag 4	Lag 5
CTI1	286511.2	0.1490	0.0162	0.0867	0.0939	0.0763
	(4.54)***	(3.25)***	(0.3499)	(1.88)*	(2.03)**	(1.66)*
CTI2	33373.10	0.2721	0.0150	0.0294	-0.0170	0.0690
	(3.69)***	(5.93)***	(0.32)	(0.62)	(-0.36)	(1.50)
CTI3	106954.0	0.1517	0.0076	0.0129	0.0020	-0.0183
	(3.94)***	(3.30)***	(0.16)	(0.28)	(0.04)	(-0.40)
CTI4	272707.4	0.1202	0.0698	-0.0281	0.0695	0.0340
	(5.07)***	(2.61)***	(1.51)	(-0.61)	(1.50)	(0.74)
Volatil	6.5020	0.0896	-0.0063	0.0173	0.0544	0.2136
	(3.54)***	(1.99)**	(-0.14)	(0.38)	(1.21)	(4.75)***

Panel B - MMI

	C	Lag 1	Lag 2	Lag 3	Lag 4	Lag 5
CTI1	3026904.	0.6320	-0.2375	0.1957	-0.0077	0.1947
	(2.89)***	(14.17)***	(-4.47)***	(3.66)***	(-0.14)	(4.36)***
CTI2	989210.1	0.4371	0.0359	-0.0123	-0.0296	0.0881
	(4.98)***	(9.65)***	(0.73)	(-0.25)	(-0.60)	(1.95)*
CTI3	83277.81	0.3713	-0.0869	0.1768	0.0679	0.1824
	(3.62)***	(8.31)***	(-1.82)*	(3.75)***	(1.42)	(4.08)***
CTI4	940759.8	0.4992	0.0935	0.0294	0.0077	0.1514
	(2.80)***	(11.11)***	(1.86)*	(0.58)	(0.15)	(3.37)***
Volatil	554.6733	0.1620	0.1405	-0.0106	-0.0253	-0.0013
	(1.02)	(3.56)***	(3.05)***	(-0.23)	(-0.55)	(-0.03)

Results of test for ARCH processes calculated by regressing the series on a constant and then regressing the squared residuals from the first regression on a constant and own lags.

***Indicates significance at the .01 level or greater
** Indicates significance at the .05 level or greater
* Indicates significance at the .10 level or greater

Table 3, Continued						
Panel C - Munis						
	C	Lag 1	Lag 2	Lag 3	Lag 4	Lag 5
CTI1	2399084.	0.2317	0.0823	0.1737	0.2103	-0.0481
	(3.90)***	(5.10)***	(1.80)*	(3.85)***	(4.61)***	(-1.06)
CTI2	242149.3	0.4438	0.0738	0.2551	0.0491	-0.0008
	(2.20)**	(9.76)***	(1.48)	(5.27)***	(0.99)	(-0.02)
CTI3	8691.690	0.1076	0.2630	0.0331	0.1495	-0.0002
	(3.39)***	(2.37)**	(5.82)***	(0.71)	(3.31)***	(-0.00)
CTI4	929024.9	0.2895	0.1238	0.3791	-0.0235	-0.0976
	(2.88)***	(6.40)***	(2.63)***	(8.58)***	(-0.50)	(-2.16)**
Volatil	0.123974	0.1363	0.0312	0.1855	-0.0207	0.2788
	(1.62)	(3.12)***	(0.71)	(4.28)***	(-0.47)	(6.39)***
Panel D - T-Notes						
	C	Lag 1	Lag 2	Lag 3	Lag 4	Lag 5
CTI1	20564130	0.27	0.1168	0.2747	0.0535	-0.1671
	(3.65)***	(6.11)***	(2.50)**	(6.06)***	(1.15)	(-3.71)***
CTI2	4258023.	0.1840	0.0898	0.3123	0.0839	-0.1269
	(3.36)***	(4.06)***	(1.95)*	(7.13)***	(1.83)*	(-2.80)***
CTI3	1384003.	0.0899	0.1448	0.0724	0.0311	0.0095
	(3.89)***	(1.97)**	(3.16)***	(1.57)	(0.68)	(0.21)
CTI4	6451468.	0.5029	0.0525	0.2002	-0.0666	-0.0625
	(3.29)***	(11.03)***	(1.03)	(3.99)***	(-1.31)	(-1.37)
Volatil	0.083030	0.3876	-0.0669	0.0799	0.0918	-0.0024
	(1.65)*	(8.48)***	(-1.38)	(1.71)*	(1.98)**	(-0.06)

Results of test for ARCH processes calculated by regressing the series on a constant and then regressing the squared residuals from the first regression on a constant and own lags.

***Indicates significance at the .01 level or greater
** Indicates significance at the .05 level or greater
* Indicates significance at the .10 level or greater

Table 3, Continued

Panel E - T-Bonds

	C	Lag 1	Lag 2	Lag 3	Lag 4	Lag 5
CTI1	470E+09	0.2115	0.0289	0.0873	0.0946	-0.0024
	(5.32)***	(4.56)***	(0.61)	(1.85)*	(2.00)**	(-0.05)
CTI2	3.11E+08	0.1937	0.1003	0.0422	0.0686	0.0121
	(5.24)***	(4.17)***	(2.13)**	(0.89)	(1.45)	(0.26)
CTI3	56204242	0.2583	0.0879	0.1594	0.0834	0.0433
	(3.44)***	(5.57)***	(1.84)*	(3.37)***	(1.75)*	(0.93)
CTI4	7.44E+08	0.1239	0.1575	0.1508	0.1795	-0.0464
	(4.20)***	(2.67)***	(3.42)***	(3.27)***	(3.90)***	(-1.00)
Volatil	0.2193	0.1350	0.1004	0.0835	0.0129	0.1538
	(2.42)**	(2.94)***	(2.17)**	(1.80)*	(0.28)	(3.37)***

Results of test for ARCH processes calculated by regressing the series on a constant and then regressing the squared residuals from the first regression on a constant and own lags.

***Indicates significance at the .01 level or greater
** Indicates significance at the .05 level or greater
* Indicates significance at the .10 level or greater

Table 4
Bivariate GARCH(1,1) Coefficient Estimates of Volatility and CTI Volume:
Silver

Coefficient	CTI1	CTI2	CTI3	CTI4
$\beta_{VOLATIL}$	1.9609 (15.25)**	1.5749 (14.39)**	1.4688 (11.10)**	2.0606 (13.44)**
β_{CTIX}	730.1503 (21.45)**	41.5286 (4.96)**	123.0743 (8.55)**	593.81 (28.23)**
Ω_{11}	0.3435 (0.89)	0.7478 (13.30)**	-0.3681 (-3.83)**	0.2120 (0.55)
α_{11}	0.6244 (14.51)**	0.5964 (19.09)**	0.5482 (16.15)**	0.2088 (3.40)**
β_{11}	0.7421 (28.55)**	0.8541 (120.45)**	0.8757 (86.02)**	0.8620 (23.50)**
Ω_{22}	3.8472 (0.00)	37.00 (11.88)**	146.4736 (8.70)**	3.8060 (0.00)
α_{22}	0.5372 (9.57)**	0.5096 (15.38)**	0.8804 (17.77)**	0.7224 (9.99)**
β_{22}	0.7251 (18.53)**	0.8859 (105.04)**	0.5315 (18.24)**	0.4805 (5.20)**
Ω_{12}	-325.1081 (-0.64)	5.3051 (1.00)	69.2430 (1.81)	-343.5159 (-0.51)
α_{12}	-10.81 (-0.95)	3.2873 (1.81)	10.3947 (3.35)**	-40.3004 (-3.82)**
β_{12}	9.2842 (1.47)	-1.0402 (-1.05)	1.1386 (0.43)	33.6280 (3.64)**
MLE	-4014.90	-3460.96	-3675.64	-3931.46

$$VOLATIL_{,t} = \beta_{VOLATIL}\, VOLATIL_{t-1} + \epsilon_{VOLATIL,\,t} \tag{2}$$
$$CTIX_{,t} = \beta_{CTIX}\, CTIX_{,t-1} + \epsilon_{CTIX,\,t} \tag{3}$$
$$\epsilon_t|(\epsilon_{t-1},..) \sim N(0,\mathbf{H}_t)$$
$$h_{11,t} = \Omega_{11} + \alpha_{11}\, \epsilon^2_{VOLATIL,t-1} + \beta_{11}\, h_{11,t-1}$$
$$h_{22,t} = \Omega_{22} + \alpha_{22}\, \epsilon^2_{CTIX,t-1} + \beta_{22}\, h_{22,t-1}$$
$$h_{12,t} = \Omega_{12} + \alpha_{12}\, \epsilon^2_{VOLATIL,t-1}\, \epsilon^2_{CTIX,t-1} + \beta_{12}\, h_{12,t-1} \tag{4}$$

Each column represents a separate set of coefficients based on the relevant CTI volume.
*Indicates coefficient is significant at .05 level.
**Indicates coefficient is significant at .01 level.

Table 5
Bivariate GARCH(1,1) Coefficient Estimates of Volatility and CTI Volume:
MMI

Coefficient	CTI1	CTI2	CTI3	CTI4
$\beta_{VOLATIL}$	0.2584 (5.82)**	0.4448 (19.13)**	0.3244 (10.19)**	0.2962 (9.06)**
β_{CTIX}	5417.3906 (48.49)**	2771.0550 (30.29)**	814.96 (36.52)**	1903.2296 (47.76)**
Ω_{11}	0.8183 (14.71)**	0.5216 (0.22)	0.7885 (12.40)**	-0.8185 (-21.81)**
α_{11}	2.7122 (23.63)**	2.8720 (44.13)**	2.7167 (30.01)**	2.5714 (32.23)**
β_{11}	0.0489 (2.90)**	0.1162 (5.42)**	0.0294 (1.77)	-0.1366 (-4.25)**
Ω_{22}	347.6201 (2.55)*	194.9004 (0.00)	58.9894 (0.64)	-195.5238 (-3.43)**
α_{22}	0.4309 (9.91)**	0.0464 (1.28)	0.4182 (11.67)**	0.6606 (12.38)**
β_{22}	-0.9005 (-54.61)**	0.6988 (0.20)	0.8927 (46.87)**	-0.7872 (-33.12)**
Ω_{12}	139.8039 (0.59)	1058.0719 (0.10)	-78.4415 (-1.27)	-223.1767 (-4.35)**
α_{12}	-23.6453 (-0.30)	0.0831 (0.01)	-5.1739 (-0.45)	39.6555 (1.71)
β_{12}	-7.7457 (-0.24)	-0.1281 (-0.05)	1.6167 (0.37)	12.2558 (1.65)
MLE	-4738.20	-4402.37	-3855.34	-4363.73

$$VOLATIL_{,t} = \beta_{VOLATIL}\, VOLATIL_{t-1} + \epsilon_{VOLATIL,\,t} \tag{2}$$
$$CTIX_{,t} = \beta_{CTIX}\, CTIX_{,t-1} + \epsilon_{CTIX,\,t} \tag{3}$$
$$\epsilon_t|(\epsilon_{t-1},..) \sim N(0,\mathbf{H}_t)$$
$$h_{11,t} = \Omega_{11} + \alpha_{11}\, \epsilon^2_{VOLATIL,t-1} + \beta_{11}\, h_{11,t-1}$$
$$h_{22,t} = \Omega_{22} + \alpha_{22}\, \epsilon^2_{CTIX,t-1} + \beta_{22}\, h_{22,t-1}$$
$$h_{12,t} = \Omega_{12} + \alpha_{12}\, \epsilon^2_{VOLATIL,t-1}\, \epsilon^2_{CTIX,t-1} + \beta_{12}\, h_{12,t-1} \tag{4}$$

Each column represents a separate set of coefficients based on the relevant CTI volume.
*Indicates coefficient is significant at .05 level.
**Indicates coefficient is significant at .01 level.

Table 6
Bivariate GARCH(1,1) Coefficient Estimates of Volatility and CTI Volume:
Munis

Coefficient	CTI1	CTI2	CTI3	CTI4
$\beta_{VOLATIL}$	0.2365 (10.70)**	0.2528 (12.53)**	0.2172 (12.99)**	0.2251 (14.43)**
β_{CTIX}	3989.8185 (35.74)**	1488.0178 (39.26)**	87.5902 (14.42)**	2237.0392 (50.24)**
Ω_{11}	0.0697 (0.58)	-0.0709 (-0.79)	0.0625 (0.99)	.0011 (-0.00)
α_{11}	0.1036 (2.72)**	0.0989 (2.41)*	0.4076 (8.72)**	0.1045 (3.73)**
β_{11}	0.8484 (9.26)**	0.8215 (13.02)**	0.7696 (22.33)**	0.8393 (38.32)**
Ω_{22}	36.2119 (0.00)	2.8465 (0.00)	0.6992 (0.00)	960.9024 (0.01)
α_{22}	0.4148 (8.22)**	0.6013 (8.65)**	0.5772 (11.75)**	0.8474 (15.71)**
β_{22}	0.3795 (0.81)	-0.8437 (-9.28)**	-0.7415 (-11.79)**	0.0474 (0.35)
Ω_{12}	2231.6257 (0.54)	461.3280 (0.38)	-66.4166 (-0.51)	275.2876 (6.92)**
α_{12}	305.1032 (0.95)	-133.9728 (-0.69)	8.3834 (0.61)	-1.6920 (-0.01)
β_{12}	121.3183 (0.11)	989.5872 (2.95)**	20.0436 (0.47)	1.5109 (0.01)
MLE	-3761.85	-3253.40	-2288.57	-3458.35

$$VOLATIL_{,t} = \beta_{VOLATIL}\, VOLATIL_{t-1} + \epsilon_{VOLATIL,\,t} \quad (2)$$
$$CTIX_{,t} = \beta_{CTIX}\, CTIX_{,t-1} + \epsilon_{CTIX,\,t} \quad (3)$$
$$\epsilon_t|(\epsilon_{t-1},..) \sim N(0,\mathbf{H}_t)$$
$$h_{11,t} = \Omega_{11} + \alpha_{11}\, \epsilon^2_{VOLATIL,t-1} + \beta_{11}\, h_{11,t-1}$$
$$h_{22,t} = \Omega_{22} + \alpha_{22}\, \epsilon^2_{CTIX,t-1} + \beta_{22}\, h_{22,t-1}$$
$$h_{12,t} = \Omega_{12} + \alpha_{12}\, \epsilon^2_{VOLATIL,t-1}\, \epsilon^2_{CTIX,t-1} + \beta_{12}\, h_{12,t-1} \quad (4)$$

Each column represents a separate set of coefficients based on the relevant CTI volume.
*Indicates coefficient is significant at .05 level.
**Indicates coefficient is significant at .01 level.

Table 7
Bivariate GARCH(1,1) Coefficient Estimates of Volatility and CTI Volume:
T-Notes

Coefficient	CTI1	CTI2	CTI3	CTI4
$\beta_{VOLATIL}$	0.2568 (5.94)**	0.2247 (10.21)**	0.1324 (14.74)**	0.1410 (9.61)**
β_{CTIX}	15958.7599 (26.90)**	6684.2297 (26.87)**	1968.4411 (30.81)**	7706.7404 (37.04)**
Ω_{11}	-0.0750 (-0.07)	-0.0552 (-0.20)	.0021 (0.00)	.0054 (0.03)
α_{11}	0.7925 (4.42)**	0.8857 (11.23)	0.8011 (14.69)**	0.1174 (5.53)**
β_{11}	0.4071 (1.65)	-1.0557 (-7.83)**	0.4901 (13.08)**	0.8903 (100.21)**
Ω_{22}	1321.2447 (0.00)	57.7531 (0.00)	1002.4009 (0.01)	760.7394 (0.00)
α_{22}	0.1265 (1.20)	0.2875 (1.79)	0.6813 (11.84)**	0.0596 (2.44)*
β_{22}	0.6281 (1.06)	0.9540 (3.75)**	0.3524 (4.04)**	0.0402 (0.05)
Ω_{12}	-6646.6062 (-0.08)	1587.0717 (0.10)	581.7073 (0.00)	4207.2753) (0.03)
α_{12}	4426.8077 (2.79)**	3276.34 (3.77)**	186.2316 (1.02)	-20.0743 (-0.08)
β_{12}	-2534.3680 (-0.68)	-6339.1545 (-3.30)**	-230.9929 (-1.34)	182.7416 (0.18)
MLE	-4164.68	-3674.95	-3277.20	-3794.36

$$VOLATIL_{,t} = \beta_{VOLATIL}\, VOLATIL_{t-1} + \epsilon_{VOLATIL,\,t} \qquad (2)$$
$$CTIX_{,t} = \beta_{CTIX}\, CTIX_{,t-1} + \epsilon_{CTIX,\,t} \qquad (3)$$
$$\epsilon_t|(\epsilon_{t-1},..) \sim N(0,\mathbf{H}_t)$$
$$h_{11,t} = \Omega_{11} + \alpha_{11}\, \epsilon^2_{VOLATIL,t-1} + \beta_{11}\, h_{11,t-1}$$
$$h_{22,t} = \Omega_{22} + \alpha_{22}\, \epsilon^2_{CTIX,t-1} + \beta_{22}\, h_{22,t-1}$$
$$h_{12,t} = \Omega_{12} + \alpha_{12}\, \epsilon^2_{VOLATIL,t-1}\, \epsilon^2_{CTIX,t-1} + \beta_{12}\, h_{12,t-1} \qquad (4)$$

Each column represents a separate set of coefficients based on the relevant CTI volume.
*Indicates coefficient is significant at .05 level.
**Indicates coefficient is significant at .01 level.

Table 8
Bivariate GARCH(1,1) Coefficient Estimates of Volatility and CTI Volume:
T-Bonds

Coefficient	CTI1	CTI2	CTI3	CTI4
$\beta_{VOLATIL}$	0.4557 (21.03)**	0.3858 (7.81)**	0.3815 (17.29)**	0.3854 (13.18)**
β_{CTIX}	251507.0949 (47.23)**	57939.5580 (44.67)**	29694.5772 (52.94)**	101482.5035 (38.27)**
Ω_{11}	-0.2071 (-9.49)**	0.0439 (0.26)	0.0230 (0.30)	-0.0018 (-0.00)
α_{11}	0.5566 (12.59)**	0.2144 (5.26)**	0.4346 (12.56)**	0.5030 (9.00)**
β_{11}	0.8984 (43.60)**	0.8454 (14.14)**	0.8551 (41.64)**	0.7396 (16.82)**
Ω_{22}	18577.7393 (2.09)*	338.2452 (0.00)	72.0663 (0.00)	25448.2992 (0.05)
α_{22}	0.5009 (6.27)**	0.5420 (6.38)**	0.4132 (6.61)**	0.4576 (4.78)**
β_{22}	0.2301 (1.52)	0.5505 (5.74)**	0.8830 (22.17)**	0.7143 (8.89)**
Ω_{12}	-71366.4228 (-12.42)**	-13512.1248 (-0.25)	-3710.0657 (-0.25)	-2160.5286 (-3.62)**
α_{12}	-7469.2483 (-0.57)**	-2831.0329 (-2.38)*	-32.6958 (-0.03)	-41.0398 (-0.01)
β_{12}	20768.5969 (2.44)*	6527.2592 (3.00)**	-41.5299 (-0.08)	-17.7230 (-0.01)
MLE	-5416.55	-4828.36	-4429.04	-5029.42

$$VOLATIL_{,t} = \beta_{VOLATIL}\, VOLATIL_{t-1} + \epsilon_{VOLATIL,\,t} \quad (2)$$

$$CTIX_{,t} = \beta_{CTIX}\, CTIX_{,t-1} + \epsilon_{CTIX,\,t} \quad (3)$$

$$\epsilon_t|(\epsilon_{t-1},..) \sim N(0,\mathbf{H}_t)$$

$$h_{11,t} = \Omega_{11} + \alpha_{11}\, \epsilon^2_{VOLATIL,t-1} + \beta_{11}\, h_{11,t-1}$$

$$h_{22,t} = \Omega_{22} + \alpha_{22}\, \epsilon^2_{CTIX,t-1} + \beta_{22}\, h_{22,t-1}$$

$$h_{12,t} = \Omega_{12} + \alpha_{12}\, \epsilon^2_{VOLATIL,t-1}\, \epsilon^2_{CTIX,t-1} + \beta_{12}\, h_{12,t-1} \quad (4)$$

Each column represents a separate set of coefficients based on the relevant CTI volume.
*Indicates coefficient is significant at .05 level.
**Indicates coefficient is significant at .01 level.