Preferences and Institutions

2.1 A General Policy Problem

Many positive analyses of economic policy share the same general structure. Consider a set of citizens facing a vector of policies $q$. This set of citizens can be small, as in a committee, or large, as in the electorate. The set is large in most policy applications. Voters are indexed by their individual attributes. We use superscript $i$ to denote variables specific to individuals of type $i$. Thus, let $\alpha$ denote the specific features of voter $i$, capturing his idiosyncratic preferences, endowments, risks, technological opportunities, or other socioeconomic attributes. The $\alpha$'s are assumed to be distributed among the citizens according to some given distribution. Individuals have utility functions defined over bundles of consumption $c$; they could also have some other well-defined economic objectives, such as profit functions.

In his role as an economic agent, an individual chooses his consumption bundle so as to maximize his utility function $U(c', q, p, \alpha')$, subject to a budget or time constraint $H(c', q, p, \alpha') \geq 0$, where $p$ is some vector of market-determined data (prices or quantities). We can thus define the indirect utility of individual $i$ as

$$\tilde{W}(p, q, \alpha') = \max U\left( (c', q, p, \alpha') \mid H (c', q, p, \alpha') \right) \geq 0$$

Any policymaker setting $q$ must respect the market-determined value of $p$ and some further constraints, such as a balanced government budget or a resource constraint that an atomistic private agent can neglect. Let us summarize these various constraints by $G(p, q) \geq 0$. Typically, the constraints will be binding, in which case the implicit function theorem allows us to write $p = P(q)$; that is, the market outcomes depend on policy and parameters.

$$\tilde{W}(q, P(q); \alpha') \equiv W(q; \alpha')$$

Using this notation, we can define the preferred policy, or the bliss point, of voter $i$ as

$$q(\alpha') = \arg \max_q W(q; \alpha')$$

Because of differences in $\alpha'$, different individuals typically have conflicting policy preferences. In this general setting, a positive analysis of economic policymaking amounts to specifying an institution and asking how it aggregates political actions, based on individual policy preferences, into equilibrium policies. Arrow’s ingenious impossibility theorem shows that no general rule enables a democracy to consistently aggregate individual preferences into policy choices. One implication of this theorem of particular interest for our purposes is that majority rule—despite its apparent predominance in real-world politics—does not generate well-defined equilibrium policies, unless we restrict its applicability either to individual policy preferences of a specific form or to political institutions of a specific type.
2.1 Restricting Preferences

The aim of this section is to study preference aggregation by pure majority rule. We define pure majority rule by the following three assumptions:

A1. Direct democracy. The citizens themselves make the policy choices.

A2. Sincere voting. In every vote, each citizen votes for the alternative that gives him the highest utility according to his policy preferences (indirect utility function) \( W(q; \alpha') \).

A3. Open agenda. Citizens vote over pairs of policy alternatives, such that the winning policy in one round is posed against a new alternative in the next round and the set of alternatives includes all feasible policies.

We will not maintain these assumptions later, when we go on to study policy choice in a representative democracy. They are useful, however, for explaining the logic of some basic theoretical results.

The Marquis de Condorcet, a French mathematician and philosopher, had pointed to the prospective problems of finding a stable outcome from majority rule already in the eighteenth century, by demonstrating how pairwise voting over policy alternatives may fail to produce a clear-cut winner. In other words, majority rule may not lead to a transitive binary relation between policy alternatives. This failure of majority rule to produce a clear winner is often referred to as the Condorcet paradox. However, we state and discuss sufficient conditions for existence of a well-defined majority winner.

2.1.1 One-Dimensional Policy

Let us make the following definition:

DEFINITION 1. A Condorcet winner is a policy \( q^* \) that beats any other feasible policy in a pairwise vote.

Suppose now that the policy space is unidimensional, so that \( q \) is a scalar. In this case, a simple way to rule out the Condorcet paradox is to use the following condition. Policy preferences defined in (2.2) are said to be single peaked for voter \( i \) if his preference ordering for alternative policies is dictated by their relative distance from his bliss point, \( q(\alpha') \): a policy closer to \( q(\alpha') \) is preferred over more distant alternatives. Specifically:

DEFINITION 2. Policy preferences of voter \( i \) are single peaked if the following statement is true:

If \( q'' \leq q' \leq q(\alpha') \) or, if \( q'' \geq q' \geq q(\alpha') \), then

\[
W(q''; \alpha') \leq W(q'; \alpha')
\]

(2.3)
We have a simple, but useful, first result:

**PROPOSITION 1.** If all voters have single-peaked policy preferences over a given ordering of policy alternatives, a Condorcet winner always exists and coincides with the median-ranked bliss point.

To prove this median-voter theorem, we can rely on a simple “separation argument.” Fix the parameter vector at some value, order the individuals according to their bliss points \( q(\alpha') \), and label the median-ranked bliss point by \( q^m \). Suppose that \( q^m \) is pitched against some other policy \( q'' \leq q^m \). By (2.3), every individual whose bliss point satisfies \( q'' \leq q(\alpha') \) prefers \( q^m \) to \( q'' \), since it is closer to his bliss point. By A2, these individuals also vote for \( q^m \). The coalition voting for supporting \( q^m \) thus constitutes a majority. Applying an analogous argument to \( q'' \geq q^m \) we obtain the result that \( q^m \) is a Condorcet winner.

Under direct and sincere voting by individuals (A1–A2) and the additional assumption of an open-agenda process (A3), we have the following:

**COROLLARY 1.** \( q^m \) is the unique equilibrium policy under pure majority rule, that is, under A1–A3.

The reason is simple: \( q^m \) beats any previous winner the first time it comes up and cannot be beaten in any subsequent vote.

From a general perspective, unidimensionality and single-peakedness are very strong assumptions. Unidimensionality of \( q \) severely restricts the available policy instruments, which may be implausible in many applications.

There are, however, more general sufficient conditions. One such general condition is the single-crossing property. An essentially equivalent condition is order-restricted preferences. Both conditions impose restrictions on the character of voter heterogeneity rather than on the shape of individual preferences. Specifically, suppose that in addition to the policy variable \( q \), the individual parameter \( \alpha \) is also unidimensional with a domain on the interval \( V \). The interval \( V \) thus denotes the set of voters. The single-crossing condition can be stated as

**DEFINITION 3.** The preferences of voters in \( V \) satisfy the single-crossing property when the following statement is true:

If \( q > q' \) and \( \alpha^i > \alpha' \), or if \( q < q' \) and \( \alpha^i < \alpha' \), then

\[
W(q; \alpha^i) \geq W(q'; \alpha^i) \Rightarrow W(q; \alpha^i') \geq W(q'; \alpha^i')
\]  

(2.4)
In other words, single crossing enables us to project preferences over \( q \) on the set of voter types, \( V \). This condition is distinct from single-peakedness but has similar implications. Specifically

**PROPOSITION 2.** If the preferences of voters in \( V \) satisfy the single-crossing property, a Condorcet winner always exists and coincides with the bliss point of the voter with the median value of \( \alpha' \).

To prove this proposition, label the critical value of \( \alpha' \) as \( \alpha^m \). Then, by (2.4), every voter with \( \alpha' \geq \alpha^m \) prefers \( q(\alpha^m) \) to any \( q < q(\alpha^m) \). Similarly, everyone with \( \alpha' \leq \alpha^m \) prefers \( q(\alpha^m) \) to any \( q > q(\alpha^m) \). In other words, \( q(\alpha^m) \) wins a pairwise vote against any conceivable alternative.

Single crossing, like single-peakedness, is capable of generating the existence of a political equilibrium under pure majority rule.

### 2.2.2 Multidimensional Policy, Unidimensional Conflict

What happens if the policy is multidimensional? From a general point of view, the existence problem gets significantly worse. Nevertheless, we can still find monotonicity restrictions on the policy preferences that guarantee the existence of a majority winner. The single-crossing property defined above can be generalized to multidimensional policies, but in that case it may be harder to verify whether it is satisfied. A simple sufficient condition can be found, however, that again ensures the existence of a Condorcet winner. This condition is less general than single crossing, but it relies on a very similar idea: voter heterogeneity is limited so that voters’ preferences for a multidimensional policy can be projected on a unidimensional space in which different voters can be ordered by their type. We label this condition “intermediate preferences”.

Specifically, let \( \mathbf{q} \) be a vector of policies and \( W(q;\alpha') \) be the policy preferences of a voter of type \( \alpha' \). As before, the parameter \( \alpha' \) is unidimensional, with a domain on the interval \( V \). Thus, although policy is multidimensional, voters differ only in one dimension, namely with regard to the parameter \( \alpha' \). Then we can state:

**DEFINITION 4.** Voters in the set \( V \) have intermediate preferences, if their indirect utility function \( W(q;\alpha') \) can be written as

\[
W(q;\alpha') = J(q) + K(\alpha')H(q),
\]

where \( K(\alpha') \) is monotonic in \( \alpha' \), for any \( H(q) \) and \( J(q) \) common to all voters.
Clearly, it is easily verified whether the voters’ preferences satisfy this condition. If so, we have a very useful result. Let \( q(\alpha^m) \) be the policy preferred by the median value of \( \alpha^i \) in the set \( V \).

Then once more:

**PROPOSITION 3.** If voters in \( V \) have intermediate preferences, a Condorcet winner exists and is given by \( q(\alpha^m) \).

The proof is again a simple separation argument, which relies on the assumed monotonicity. As \( q(\alpha^m) \) is a maximum, we have that

\[
K(\alpha^m) \leq \frac{J(q(\alpha^m)) - J(q)}{H(q) - H(q(\alpha^m))} \quad \text{as} \quad H(q) > H(q(\alpha^m)),
\]

for any \( q \neq q(\alpha^m) \). Furthermore, voter \( i \) supports \( q(\alpha^m) \) against \( q \) if

\[
K(\alpha^i) \leq \frac{J(q(\alpha^m)) - J(q)}{H(q) - H(q(\alpha^m))} \quad \text{as} \quad H(q) > H(q(\alpha^m)),
\]

As \( K(\alpha^i) \) is monotonic in \( \alpha^i \), the first condition implies that the second is fulfilled for at least half the voters. The policy vector \( q(\alpha^m) \) collects at least half the votes against any alternative policy.

Intuitively, the intermediate preference property allows us to project the conflict in a multidimensional policy space into a unidimensional parameter space, where we can apply a separation argument as before. Essentially, this can occur only when there is a single source of disagreement among different individuals.

In a general theoretical perspective, these restrictions on preferences may appear very stringent. This is indeed the message from social choice theory and spatial voting theory. But both of these theories start their analysis by directly formulating the voters’ preferences in some general outcome space. Applied analyses of economic policymaking instead generally derive agents’ preferences over policy from assumptions about the economic environment. That is, they make assumptions about the usual primitives in economic analysis: agents’ utility functions, technologies, and endowments and the available policy instruments. The intermediate-preference condition then turns out to be fulfilled in some nontrivial examples. In this applied perspective, the necessary assumptions may not appear considerably more restrictive than the assumptions in macroeconomics, public finance, or contract theory. Interestingly, in their historical analysis of congressional votes in the United States it was found that the voting pattern of U.S. congressmen
can largely be aligned along a single dimension of disagreement that can be interpreted as left to right in the traditional ideological sense.

2.3 Restricting Institutions

We have seen that meaningful restrictions on preferences can be imposed that guarantee the existence of a unique equilibrium under pure majority rule as we have defined it in A1–A3. If the conditions imposed by these restrictions are not met, a Condorcet winner generally does not exist. In that event, a number of problems arise under pure majority rule: an open agenda may lead to infinite voting cycles, incentives for agenda manipulation arise, and agents have strong incentives to vote strategically, rather than sincerely. We now turn to a discussion of these issues.

2.3.1 Nonexistence of a Condorcet Winner and Its Implications

In the case of a genuine multidimensional policy conflict, the conditions for existence of a Condorcet winner indeed become very strong. This has been forcefully demonstrated in the context of the multidimensional spatial voting model, which starts out from an a priori formulation of agents’ preferences. In the notation above, these preferences have the general form $W(q - \alpha')$, where $q$ is a policy vector and $\alpha'$ a vector of the coordinates describing voter $i$’s bliss point in this policy space. Also, $W$ is assumed to be decreasing and concave—typically symmetric and often spherical—in the distance $||q - \alpha'||$. Empirical results demonstrate that the condition required for a point $q^*$ to be a Condorcet winner is very strong. In particular, a hyperplane cutting through $q^*$ in any direction must divide the set of individual bliss points into two subsets with an equal number of voters. The condition boils down to the existence of a “median in all directions,” allowing for a separation argument like those applied in the previous section. A priori, such extreme symmetry in the distribution of individual bliss points seems an utterly unlikely occurrence, even though the intermediate-preference property discussed above implies that the individual bliss points lie along a continuous and monotonic curve in the policy space.

Many economic policy problems do generate policy preferences of such form. Nonexistence of a majority winner is particularly likely for policies that can be targeted to individual groups of voters. Many types of redistributive programs obviously fall into this category. In discussing the consequences of nonexistence of a majority winner, we consider a very simple example of targeted redistribution, namely, the so-called pie-splitting problem.

Example 5. Pure redistribution. Consider redistribution of a fixed amount among three voters. Voter $i, i = 1, 2, 3$, thus has preferences

$W(q) = U(q_i)$,
where $U$ is a common concave utility function and $q'$ is a nonnegative transfer out of a fixed budget normalized to unity:

$$\sum_{i=1}^{3} q'_i = 1.$$ 

By this budget constraint, the policy problem is two-dimensional. The three voters' policy preferences can thus easily be illustrated in a diagram drawn in $(q^1, q^2)$ space with $q^1$ on the horizontal axis and $q^2$ on the vertical axis, as in figure 2.1.

In the figure, voter 1’s most preferred point is (1, 0), where he gets the whole available pie, whereas the most preferred points of voters 2 and 3 are (0, 1) and (0, 0), respectively. Furthermore, we can draw the indifference curves of voter 1 as straight vertical lines, those of voter 2 as straight horizontal lines, and those of voter 3 as straight lines with a slope of $-1$.

To illustrate how the open-agenda process, defined in A3, may lead to cycling, consider a simple case with only three exogenously given policy vectors, indexed by $q_a$, $q_b$, and $q_c$ in figure 2.1.

Suppose a vote is first taken on $q_b$ versus $q_c$. Both voters 1 and 2 prefer $q_b$, which lies on a higher indifference curve than $q_c$ (they split the losses of 3). When $q_b$ is pitched against $q_a$ in the next round, voters 1 and 3 prefer $q_a$. Yet when $q_a$ is posed against $q_c$, $q_c$ collects a majority made up by voters 2 and 3. As $q_c$ is then posed against $q_b$, the same cycle starts again and can go on forever.
What if the agenda is restricted, so as to include voting only in a finite number of steps? Even though the restriction guarantees that an end point will be found, this does not, by itself, help us a great deal in making predictions about the policy outcome, for the set of possible outcomes is still very large. It has been shown that, with spatial (spherical) preferences, a sequence of pairwise votes connects any starting point with any possible outcome in the Pareto set. The latter is defined as the set of points at which no voter can be made better off without making another voter worse off; in the simple pure-redistribution example above, it is illustrated in figure 2.1 by the isosceles triangle connecting the bliss points of the three voters.

But if the agenda is restricted, these results clearly imply that whoever controls the agenda can use it to his own advantage. As an illustration, consider again the three alternatives $q_a$, $q_b$, and $q_c$ in figure 2.1 and assume that only two rounds of pairwise voting can take place. Suppose that voter 1 sets the agenda. The optimal procedure from his point of view is to start the vote by pitching $q_b$ versus $q_c$. This selects $q_b$, which is then bound to lose against $q_a$, which is the alternative favored by voter 1. In the same vein, voter 2 (3) can implement his preferred alternative $q_b$ ($q_c$), by posing $q_a$ against $q_c$ ($q_a$ against $q_b$) in the first round. Restricted agendas thus open the door for strategic agenda manipulation. Notice, however, that it is the restricted-agenda assumption and not the nonexistence of a Condorcet winner that leads to the incentive for agenda manipulation. In other words, agenda setting may be valuable even if a Condorcet winner exists.

So far in this chapter we have relied on assumption A2, that agents vote sincerely at every stage. The forces that render agenda manipulation profitable make that assumption questionable, however, as they also provide strong incentives for strategic voting. To see this, suppose that voter 1 sets the agenda, so as to achieve his preferred point $q_a$ in the example of figure 2.1. Assume also that voters 1 and 3 continue to vote sincerely at both stages. Then voter 2 can improve his situation by voting strategically at stage 1, because voter 2 is pivotal at that stage. By voting for his nonpreferred alternative $q_c$ at stage 1, voter 2 ensures that this alternative wins not only at stage 1, but also at stage 2 when posed against $q_a$ (nobody can be better off by voting strategically at the last stage). Clearly, this outcome is better for voter 2, whose share of the pie is larger in $q_c$ than in $q_a$. Generally, foresight of the outcome at future stages of voting effectively makes the voters face more than two alternatives at the first stage.

Are the incentives for strategic voting an artifact of the special assumptions behind this simple example? No. They are inherent to almost any voting situation involving more than two alternatives. Any democratic decision-making process, including majority rule, involving three or more alternatives is open to strategic preference manipulation. This means that sincere voting (A2) is not an attractive assumption if an open-agenda process (A3) does not imply convergence to a Condorcet winner.
2.3.2 Modeling Institutions of Policy Choice

As we have seen, searching for a universally applicable theory of political equilibrium is a futile exercise. Furthermore, majority voting will generically lead to cycles, unless the voting agenda was restricted. Such restrictions, however, give strong incentives to strategic manipulation, either of the agenda itself or of the preferences revealed in the voting process. Any positive theory of political choice—whether it was based on majority rule or not—seemingly has to rely on unattractive or arbitrary assumptions. At the same time, the cycling and instability that existing models suggest do not seem to be a feature of democratic decision making in practice. New ideas sowed the seeds of a more optimistic view. These ideas include the probabilistic voting model, the structure-induced equilibrium model, and the agenda-setter model.

These ideas stem from a common premise: policy choices are not made by the citizens themselves under direct democracy, but are delegated to elected representatives. Assumption A1 is thus relaxed, appealing to the organization of decision making in real-world democracies. The details of how policy is chosen impose additional structure on the political process that, in turn, can give rise to a well-defined equilibrium. Each of these approaches identifies a specific aspect of political institutions as a crucial determinant of policy. Probabilistic voting is a theory of electoral competition in which politicians offer policy platforms to the voters and specific assumptions are made about the voters’ behavior. Structure-induced equilibrium and the agenda-setter model apply to collective decisions in smaller groups of political representatives, like a committee or a legislature, in which representatives have well-defined policy preferences and the institution imposes a particular procedure for decision making. Thus in these models the policy decision is the outcome of a game with a well-defined extensive form. In the following, we briefly introduce the main ideas behind each of these approaches.

Probabilistic Voting The traditional starting point for analyzing electoral competition was the classical theory of Hotelling and Downs. By the 1970s, formal results had verified the main insight of these authors. Suppose, as did Hotelling and Downs, that elections involve two identical politicians (or parties). The politicians are opportunistic in the sense of being purely office-motivated: they strive to maximize their vote share or, alternatively, the probability of winning. Moreover, these politicians can make binding commitments to policy platforms in the course of the electoral campaign. The outcome is a Condorcet winner, if such a policy exists. The intuition is very simple and closely related to the separation arguments in section 2.2. Faced with only two policy alternatives (the two electoral platforms), all citizens vote sincerely. Thus a policy platform coinciding with a Condorcet winner always captures at least half of the vote when it is up against any other platform. Consequently, the situation in which both candidates select the policy preferred by the pivotal voter is the only one where no candidate (party) can discontinuously increase his probability of winning (its vote share). As the candidates are office-motivated, this is, by definition, Nash equilibrium.
With multidimensional policy conflict, on the other hand, Downsian electoral competition games generally do not have any equilibria. This is a direct consequence of the cycling problems discussed above. If no policy dominates any other policy, one candidate can always find another policy that is preferable for a majority of the voters, given any policy platform proposed by the other candidate. The objective functions of the office-seeking candidates thus become highly discontinuous throughout the policy space.

Probabilistic voting models essentially smooth out these objectives by introducing uncertainty—from the candidates’ viewpoint—about the mapping from policy to aggregate voting behavior. The argument comes in several guises, with different degrees of micropolitical foundations. Individual voters may abstain from voting if the proposed policies are too far away from their ideal points or if not too much is at stake. Or the candidates may perceive the probability that a particular voter (or group of voters) votes for a particular candidate as a continuous function, rather than a step function, of the distance between the two platforms. In either case, the expected number of votes becomes a smooth function of the policy platform, which guarantees existence of a Nash equilibrium under some regularity conditions on the underlying utility and distribution functions.

To illustrate these points more precisely, consider a policy choice like the one discussed in example 3. Thus two types of public consumption, \( q_1 \) and \( q_2 \), are financed by general taxation. As before, \( q_1 \) and \( q_2 \) are the policy variables with taxes residually determined. There are \( I \) voter types, \( i = 1,..., I \) (with \( I \) a large number), who evaluate policy according to preferences \( W(q;\alpha) \).

But now these preferences do not fulfill the monotonicity property imposed in example 3, meaning that an open-agenda process over pairs of policy vectors could give rise to cycling. Two parties, \( A \) and \( B \), simultaneously announce their policy platforms ahead of the election, \( q_A \) and \( q_B \) respectively. The party winning the election implements his promised policy, and parties maximize the probability of winning.

Let \( \pi_i^p \) be the probability perceived by the candidates that voter \( i \) votes for party \( P \), \( P = A, B \), and suppose that these probabilities refer to independent events for different voters.

Then the expected vote share of party \( P \) is

\[
\pi_p = \frac{1}{I} \sum_{i=1}^{I} \pi_i^p.
\]

Under Downsian electoral competition with two identical parties, \( \pi_i^p \) jumps discontinuously from 0 to 1 as voter \( i \) always votes with certainty for the party that promises the better policy.

Because of these discontinuous jumps, a Nash equilibrium in the electoral competition game may fail to exist. One way or the other, probabilistic voting models instead assume that
\( \pi_A^i = F^i \left( W \left( q_A^i; \alpha^i \right), W \left( q_B^i; \alpha^i \right) \right) \), where \( F^i (\cdot) \) is a smooth and continuous function, increasing in the first argument and decreasing in the second. This smoothness implies that a small unilateral deviation by one party does not lead to jumps in its expected vote share and thus gives rise to well-defined equilibria.

An interesting special case restricts these probabilities to take the form
\[
\pi_A^i = F^i \left( W \left( q_A^i; \alpha^i \right) - W \left( q_B^i; \alpha^i \right) \right),
\]
where \( F^i (\cdot) \) is a continuous and well-behaved cumulative distribution function (c.d.f.), associated with a probability distribution. Furthermore, suppose that parties maximize their expected vote share. In this case, party \( A \) sets \( q_A \) to maximize:
\[
\pi_A = \frac{1}{I} \sum_{i=1}^{I} F^i \left( W \left( q_A^i; \alpha^i \right) - W \left( q_B^i; \alpha^i \right) \right).
\] (2.10)

Clearly, party \( B \) faces a symmetric problem, and in a Nash equilibrium with simultaneous policy announcements both candidates announce the same equilibrium policies: \( q_A = q_B \). Moreover, the first-order conditions for a maximum of (2.10), evaluated at the equilibrium policy \( q_A \), and taking \( q_B \) as given, can be written as
\[
\sum_{i=1}^{I} f^i (0) W_{q1\alpha} (q_A^i; \alpha^i) = 0
\]
\[
\sum_{i=1}^{I} f^i (0) W_{q2\alpha} (q_A^i; \alpha^i) = 0.
\]

In these expressions, \( f^i (0) \) denotes the density corresponding to the c.d.f. \( F^i (\cdot) \), evaluated at 0 (namely in equilibrium). Thus the equilibrium under this form of electoral competition implements the maximum of a particular weighted social welfare function, where voter \( i \) receives weight \( f^i (0) \). Voters with higher \( f^i (0) \) weigh more heavily, because in a neighborhood of the equilibrium they are more likely to reward policy favors with their vote. That is, more “responsive” voters, who have a higher density \( f^i (0) \), receive a better treatment under electoral competition. Clearly, if all voters are equally responsive (if they all have the same value of \( f^i (0) \)), this form of electoral competition implements the utilitarian optimum.

**Structure-Induced Equilibrium** The next model of collective choice, structure-induced equilibrium, disregards elections. Instead it analyzes the decisions by a group of representatives, with given policy preferences, which is in charge of making policy decisions in a committee or a legislature. The political institution prescribes some procedure for reaching a consensus. Specifically, consider a situation in which the decision can be split in different stages, each stage being under the jurisdiction of a specific committee or being the outcome of a separate vote. The
specific assumptions are most easily illustrated in the context of a concrete example. Reconsider therefore example 3, in which policy consists of two types of public consumption, \( q_1 \) and \( q_2 \), financed by general taxation. Policy choice is delegated to a legislature with three members \( i \) — we can interpret these as three parties, representing three groups of citizens—who evaluate policy according to preferences \( W(q; \alpha') \). These preferences do not, however, fulfill the monotonicity property imposed in example 3, so that an open-agenda process over pairs of policy vectors gives rise to cycling. Figure 2.2 illustrates examples of such preferences: the legislators’ most preferred policies are given by the points \( q(i \alpha') \), \( i = 1, 2, 3 \), with surrounding elliptic indifference contours.

Imagine that the decisions on each publicly provided good are made in an open-agenda process in which legislators vote separately and sequentially over each dimension. First decisions are made over, say, \( q_1 \), then over \( q_2 \) for a given \( q_1 \). All alternatives are compared pairwise in each dimension separately. This approximates the legislative practice of letting two different committees handle the two types of public consumption and only allowing the legislature to consider amendments under the jurisdiction of one committee at a time. A crucial assumption is that all legislators vote sincerely.

Consider the last stage, in which a vote is taken over \( q_2 \) for a given \( q_1 \). As figure 2.2 is drawn, voter 1 is the median voter. He is constrained to pick a point along the vertical line corresponding to the value of \( q_1 \) that was selected at the first stage. He thus selects the tangency point between the vertical line and his indifference curve. As we vary \( q_1 \), we thus trace out voter 1’s “reaction function,” the locus of points where the indifference curves of voter 1 have vertical slope. Consider now the first stage, at which a vote over \( q_1 \) is taken. Here, as figure 2.2 is drawn, the median voter is voter 3, who realizes that the final equilibrium will be a point on voter 1’s reaction function. Voter 3 can choose which point by selecting a value of \( q_1 \). His best choice is obviously a point where voter 1’s reaction function is just tangent to his own indifference curve, point A in figure 2.2.
This simple outcome is valid if legislator preferences obey a “single-crossing condition in each direction,” such that legislator 3 is always pivotal in the decision over $q_1$, whatever the level of $q_2$, and legislator 1 is always pivotal in the decision over $q_2$. If this regularity condition is not fulfilled, an equilibrium still exists, but the identity of the pivotal voter for each public good depends on the given supply of the other. Note that as we reverse the order of votes, the equilibrium changes to point B, where voter 3’s reaction function is tangent to voter 1’s indifference curve: point B in figure 2.2. Not surprisingly, the order of voting decisions matters for the final outcome.

**Agenda Setting**  As indicated at the beginning of this subsection, agenda setting was associated with agenda manipulation in situations of cycling, which in turn was associated with the failure to find a universally applicable method of aggregating individual preferences into political decisions. In many political decisions, however, specific politicians or bureaucrats do have a great deal of influence on the alternatives the decision makers face: they may not only have the power to propose, but also to prevent amendments from being made (gatekeeping power), such that a closed-agenda process is a better description of reality. Successful positive modeling of political decisions will then have to take these powers into account. The same argument applies, of course, irrespective of whether the policy issue at hand is one-dimensional and a Condorcet winner exists. Suppose it does. Then if the agenda setter’s preferred policy does not coincide with the pivotal voter’s, neither will the equilibrium policy. It has been showed that an agenda-
setting model empirically outperformed a median-voter model in explaining decisions on spending in Oregon school districts made by referenda.

As the discussion in section 3.1 suggests, however, assuming a closed agenda may also be a way to cut through the nonexistence problem in multidimensional policy examples. We now illustrate this possibility, and some additional implications of the agenda-setting model, in a slight modification of our earlier pure redistribution example.

Consider again the pie-splitting problem. The preferences of the three groups of voters are the same. But instead of voting on an exogenous set of alternatives, as before, we now let one of the groups \( i = A \) (or a legislator representing that group) make a policy proposal, \( q_A = (q_1^A, q_2^A, q_3^A) \).

For simplicity, we consider the simplest form of a closed agenda. There is thus only one round of voting, no amendments, and the (single) policy proposal pitched against a default policy: the status quo \( \bar{q} = (\bar{q}_1, \bar{q}_2, \bar{q}_3) \). If the proposal \( q_A \) collects a simple majority—the vote of two groups—it goes through, \( q = q_A \); if not, the status quo is implemented, \( q = \bar{q} \). Clearly, the proposer gets support from another group only if the latter is offered a share of the pie that exceeds its share in the status quo, such that \( U(q_A^i) \geq U(\bar{q}^i) \) for at least one \( i \neq A \).

The equilibrium outcome of this majoritarian “ultimatum game,” when each of the groups has the power to propose, illustrates two important and general results. First, it illustrates the principle of minimum winning coalitions. When setting the agenda, each group seeks the support of only one other group: giving both the other groups at least their status quo allocations would be a waste of resources. Thus, the equilibrium allocation must always give zero to one player.

Second, it shows that any agenda setter seeks majority support in the cheapest way, offering the status quo amount \( \bar{q}_A \) (plus epsilon to break the tie) to the group with the worst status quo outcome. A weak bargaining position in this sense is thus beneficial, because it raises the likelihood of being included in the winning coalition and getting at least the status quo outcome. This contrasts with the properties of a regular two-person bargaining game, in which both players must agree and having more bargaining power is a good thing.

This kind of model has, in fact, been extended into a theory of noncooperative legislative bargaining games, in which different games in extensive form capture different rules for legislative decisions. It has been showed that equilibria always exist, even in infinitely repeated bargaining under open agendas, if one assumes that delay in coming to an agreement is costly.

**The Role of Elections** The three models of politics just discussed (probabilistic voting, structure-induced equilibrium and agenda setting) all presuppose some form of representative democracy, where policy choices are delegated to political representatives. But they differ in one fundamental respect. Probabilistic voting is a model of electoral competition where two competing candidates commit to specific and detailed policy promises before the elections. It
belong to the world of preelection politics. The last two models, on the other hand, describe policy formation within a legislature that is not bound by a precise electoral promise. They thus belong to the world of postelection politics. The more recent literature has added an electoral stage, at either the end or at the beginning of the policy formation stage, in which political representatives compete against an opponent. Elections thus play a role in both type of models, but the role is very different.

When political candidates can commit to a policy ahead of the elections, all the action with regard to policy choice takes place when the candidates locate themselves in the policy space. For this reason we refer to models of electoral competition as models of preelection politics. Theories of preelection politics are similar to models of location in economics (Hotelling), or to models of final-offer arbitration, in which two parties in a dispute submit final offers to an arbitrator who, in turn, decides which offer wins. Here the voters are like the arbitrator and select among alternative policies. In this sense, elections truly aggregate the voters’ policy preferences. Implicit in this role of elections is a populist view of democracies, namely the view that well-functioning democratic institutions really allow citizens to express their preferences among alternative policies.

In models of legislative bargaining, on the other hand, candidates cannot commit to policies in advance of the elections. In such models the voters do not select among alternative policies, but rather among alternative agents to be appointed and play the policy formation game. That is, elections are a way of monitoring an incumbent’s behavior or to select the most appropriate representative, either in terms of talent or ideology. In this alternative perspective, nonexistence of a Condorcet winner is not such a dramatic problem: the rules and constraints on the policy formation game often give rise to a well-defined equilibrium, even if the voters’ preferences over policies are not well behaved. In this view of politics, a political constitution is a bit like an incomplete contract. Political representatives are appointed not on the basis of their policy promises, but on how they will play or have played the policy formation game. Elections allow voters to assign or remove decision-making rights, and who gets elected may be important precisely because there is a fundamental incompleteness in the electoral mandate. As mentioned in chapter 1, we refer to this second strand of literature as postelection politics, because all the relevant policy action takes place once a candidate is in office, not before. This line of research is more attuned to a liberal view of democracies, namely the view that the role of democratic institutions is to remove from office representatives who have not pursued the interests of their citizens. From the point of view of economic theory, theories of postelection politics borrow many tools and ideas from models of agency.