

NAME: \_\_\_\_\_

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**Exam 2 - MAC 2311**

Spring 2013

**General Directions:** *Read the problems carefully and provide answers exactly to what is requested. Use complete sentences and use notation correctly. Incomprehensible work is worthless. Full credit will be awarded only for work which is both correct and neatly presented. I am grading the work, not just the answer. Don't rush, don't try to do too many steps of a computation at once; work carefully. Good luck!*

1. (30 pts) Find  $dy/dx$ . Simplify when possible (6 pts each):

(a)  $y = \frac{x^3}{3} - 2\sqrt{x} + \frac{3}{2}$

(b)  $y = x^2 e^{-x}$

(c)  $y = \sqrt{1 + \sin^2 x}$

(d)  $y = \arctan(x) + \arctan\left(\frac{1}{x}\right)$

(e)  $y = (x^2 + 1)^x$

2. (10 pts) Prove the formula for  $(\cot x)'$ . You may use the derivative formulas for  $\sin x$  and  $\cos x$  functions.

3. (10 pts) The function  $h(x)$  is given by  $h(x) = \frac{x^2}{f(x)}$ . It is known that  $f(2) = 1$  and  $f'(2) = 3$ .

Compute: (a) (3 pts)  $h(2)$

(b) (7 pts)  $h'(2)$ .

4. (10 pts) Use the limit definition of the derivative to find  $f'(x)$  if  $f(x) = \frac{1}{x}$ .

5. (14 pts) The adjacent graph represents the population function  $P(t)$  for yeast cells in a laboratory culture.

(a) (6 pts) Based on the graph, fill in the values of the following limits

$$\lim_{t \rightarrow +\infty} P(t) = \text{_____} \qquad \lim_{t \rightarrow +\infty} P'(t) = \text{_____}$$

(b) (8 pts) Approximately when is the population of yeast cells increasing the fastest? Estimate  $dP/dt$  at that moment. Give units to your answer.

6. (10 pts) Show that for any constants  $A, B$ , the function  $y = Ae^{-3x} + Be^{2x}$  satisfies  $y'' + y' - 6y = 0$ .

7. (10 pts) An automobile is driven down a straight highway such that after  $0 \leq t \leq 8$  seconds, it is  $s = 5t^2$  feet from its initial position.

(a) Find the average velocity of the car over the interval  $[0, 8]$ . Give units to your answer.

(b) Find the instantaneous velocity of the car at  $t = 6$ . Give units to your answer.

8. (16 pts) (a) (10 pts) Find the equation of the tangent line to the curve  $x^2 + 2xy + 4y^2 = 12$  at the point  $(2, 1)$ .

(b) (6 pts) Find all points on the curve  $x^2 + 2xy + 4y^2 = 12$  where the tangent line is horizontal.