## Worksheet week 3

1. Compute each of the following limits. If the limit does not exist or is infinite, specify so.
(a) $\lim _{x \rightarrow 0} \frac{1-\cos (3 x)}{x^{2}}$
(b) $\lim _{x \rightarrow+\infty} \frac{1-\cos (3 x)}{x^{2}}$
(c) $\lim _{x \rightarrow 0} \frac{x \sin (5 x)}{\tan ^{2}(3 x)}$
(d) $\lim _{x \rightarrow+\infty} \sin \left(\frac{\pi x}{2 x+1}\right)$
2. (a) Use the Intermediate Value Theorem (IVT) to approximate all real roots of the equation $x^{3}-4 x^{2}+1=0$ to within intervals of length 1 . Can you do the same but within intervals of length 0.5 ?
(b) From IVT it actually follows that any cubic equation has at least one real solution. Can you briefly explain why is this so?
