MAC 2311: Worksheet June 18, 2018 **Group nr:** _____

Applications of the completion of the square

Problem 1. A farmer plans to enclose a rectangular piece of land on three sides (the fourth side is along a river and does not need fencing). If the farmer has 500 ft of fencing available, what are the dimensions of the plot that with the largest area?

Problem 2. A farmer plans to enclose a rectangular piece of land on three sides (the fourth side is along a river and does not need fencing). This time it is required that the area of the plot is 20,000 ft^2. What dimensions should the farmer choose for the plot in order to minimize the amount of fencing used.

Note: Both problems 1 and 2 are optimization problems, that is they involve finding maximum or minimum of certain functions. Completion of the square works for problem 1, because the involved function is quadratic, and it also works in Problem 2, with a bit of tricky algebra. However, for most optimization problems, completion of the square will work. Calculus will give us an efficient and general way of solving such problems.

Rates of change

Problem 3. A stone is thrown straight upwards from the ground. Its height (in feet) above the ground t seconds after it is thrown is given by $s(t) = -16t^2 + 96t$.

(a) When does the stone land back on the ground?

(b) Compute the average velocity of the stone during the first two seconds.

(c) Compute the instantaneous velocity of the stone at $t = 2$ s.

(d) Sketch the graph of the function $s(t) = -16t^2 + 96t$ and explain how your answers from parts (b) and (c) are related to this graph.

Problem 4. (a) Sketch the graph of $y = f(x) = x^2 - 2x$.

(b) On your graph, draw three lines whose slopes will correspond to each of the following rates of change of f:
(i) the average rate of change of f on the interval $[1, 2]$;
(ii) the average rate of change of f on the interval $[2, 3]$;
(iii) the instantaneous rate of change of f at $x_0 = 2$.
Based on your picture, which of these is the largest?

(c) Compute algebraically the rates of change from (i), (ii) and (iii) and confirm your answer of part (b).

(d) Compute the instantaneous rate of change of f at an arbitrary point x_0.