Group nr. ______________________ **NAMES:** ______________________

MAC 2311: Worksheet 7/02/2018 (Rules II, Derivatives of Trig. Functions)

LECTURE INTRO: Compute derivative of $\sin(x)$ and give the one for $\cos(x)$. (Mention that x is in radians.) Derive the product law and give the quotient law.

1) Compute the following derivatives:

a) $\frac{d}{dx}(x^3 \sin(x))$

b) $\frac{d}{dx}(3\sin^2(x))$

c) $\frac{d}{dx}\left(\sqrt{x}(x^2 - 7x)\right)$

d) $\frac{d}{dx}\left(\frac{x^2+1}{\sqrt{x}+3}\right)$

2) Using your knowledge of the derivatives of $\sin(x)$ and $\cos(x)$ and of the product and quotient laws, compute the following derivatives:

a) $(\tan(x))'$

b) $(\cot(x))'$

c) $(\sec(x))'$

d) $(\csc(x))'$

3) Compute the following derivatives:

a) $\frac{d}{dx}\left(5 + \frac{1}{\tan(x)}\right)$

b) $\frac{d}{d\theta}\left(\frac{\sec(\theta)}{1+\sec(\theta)}\right)$

c) $\frac{d}{dt}\left((\sin(t) + \cos(t))\csc(t)\right)$

4) Show that $y = x\sin x$ is a solution to the differential equation $y'' + y = 2\cos x$.

5) The curve $y = \frac{x}{1+x^2}$ is sometimes called a "serpentine" (you can check the graph on a graphing calculator or on wolframalpha.com to see why).

(a) Find the equation of the tangent line to the curve at $x = 0$.

(b) Find the coordinates of the points where the tangent line to the serpentine is horizontal.

6) The following provides a proof for the quotient rule from the product rule.

Let $q(x) = \frac{f(x)}{g(x)}$, be the quotient of two functions $f(x)$ and $g(x)$.

The goal is to get a formula for $q'(x)$ in terms of $f'(x), g'(x), f(x), g(x)$. Proceed as follows:

Start from $q(x) \cdot g(x) = f(x)$. (Why is this true?)

Take the derivative of both sides of the above and use product rule on the left side. Then solve for $q'(x)$ and do a bit of algebra to eventually get the familiar quotient rule formula.