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Exam 2-MAC 2311

## Important Rules:

1. Unless otherwise mentioned, to receive full credit you MUST SHOW ALL YOUR WORK. Answers which are not supported by work might receive no credit.
2. Please turn your cell phone off at the beginning of the exam and place it in your bag, NOT in your pocket.
3. No electronic devices (cell phones, calculators of any kind, etc.) should be used at any time during the examination. Notes, texts or formula sheets should NOT be used either. Concentrate on your own exam. Do not look at your neighbor's paper or try to communicate with your neighbor.
4. Solutions should be concise and clearly written. Incomprehensible work is worthless.
5. (36 pts) Find $d y / d x$ ( 6 pts each). You do not have to simplify, except in part (c).
(a) $y=4 \arcsin x+\frac{2}{\sqrt{x}}+\pi^{2}$
(b) $y=\frac{x^{3}+\sqrt[3]{x}}{3^{x}}$
(c) $y=\arctan \left(e^{x}\right)+\arctan \left(e^{-x}\right) \quad$ Note: 2 bonus points if you fully simplify your answer here.
(d) $y=\sec ^{2}\left(x^{3}\right)$
(e) $y=\sqrt{1+(\tan x)^{-2}}$
(f) $y=\left(x^{2}+1\right)^{1 / x}$
6. (16 points) These are True or False questions. Circle your answer AND give a brief justification.
(a) If $y=\cos (f(x))$, then $d y / d x=-\sin \left(f^{\prime}(x)\right)$. True False

## Justification:

(b) If $g(x)=f(x) \sin x$ then $g^{\prime}(0)=f(0)$. True False

## Justification:

(c) If a function $f(x)$ is differentiable at $x=2$ then it is continuous at $x=2$. True False

## Justification:

(d) If $p(x)$ is a polynomial of degree 10 , then its eleventh derivative, $p^{(11)}(x)=0 . \quad$ True $\quad$ False

## Justification:

3. ( 10 pts ) A rocket, rising vertically, is tracked by a radar station that is on the ground 10 miles from the launchpad. How fast is the rocket rising when it is 8 miles high and its distance from the radar station is increasing at a rate of 1 mile/second ?
4. (10 pts) Show that $y=x \cos (2 x)$ is a solution for the differential equation $y^{\prime \prime}+4 y=-4 \sin (2 x)$.
5. (12 pts) A manufacturer of athletic footwear finds that the sales of their Xtride brand of running shoes is a function $f(p)$ of the selling price $p$ (in dollars) for a pair of shoes. Suppose that $f(120)=5000$ pairs of shoes and $f^{\prime}(120)=-50$ pairs of shoes per dollar. The revenue that the manufacturer will receive for selling $f(p)$ pairs of shoes at $p$ dollars per pair is $R(p)=p \cdot f(p)$.
(a) (8 pts) Find $R^{\prime}(120)$.
(b) (4 pts) In one sentence, explain the practical meaning of the result in part (a).
6. (14 pts) (a) (8 pts) Use implicit differentiation to find $d y / d x$ for the (rotated) ellipse $x^{2}-x y+3 y^{2}=11$.
(b) ( 6 pts ) Find all points on the ellipse $x^{2}-x y+3 y^{2}=11$ where the tangent line is horizontal.
7. (10 pts) Choose ONE:
(a) State and prove the formula for the derivative of a product of two functions.
(b) Find, with proof, the formula for $(\cos x)^{\prime}$.
