

Name: _____

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Optimization Worksheet

Calculus I – Spring 2016

General steps of solving optimization problems:

- (a) Identify what quantity you are trying to optimize.
- (b) Draw a picture. Label variables and indicate eventual constants.
- (c) Express the variable to be optimized as a function of the variables you used in part (b).
- (d) Find relations among the variables from (b) and express the variable to be optimized as a function of just one of the variables from (b).
- (e) Identify the domain for the function you found in part (d).
- (f) Find the absolute maximum of the variable to be optimized on this domain.
- (g) Reread the question and be sure you have answered exactly what was asked.

1. The boundary of a field is a right triangle with a straight stream along its hypotenuse and with fences on its other two sides. Find the dimensions of the field with the maximum area that can be enclosed using 1000 feet of fencing. Solve this problem by carrying out the following steps:

2. You are designing a poster that will contain 50 square inches of text, which must have a 4 inch margin at the top and bottom and a 2 inch margin on the left and right sides. What overall dimensions will minimize the amount of paper that needs to be used?

3. A square-based shipping crate is being designed that must contain a volume of 16 ft^3 . The material that is used for the base and the lid costs 3 dollars/ ft^2 , while the material used for the sides costs 2 dollars/ ft^2 . What are the most cost-effective dimensions of such a crate?

4. What are the dimensions of the lightest cylindrical can that can hold a given volume of $V_0 \text{ cm}^3$? Assume the same material is used for the lid, the base and the side of the can. [Hint: the lightest can is the can with the smallest surface area.]

5. Suppose that a tour service offers the following rates:

- \$200 per person if 50 people go on the tour (this is also the minimum number of people needed to book the tour).
- For each additional person over 50, up to a maximum of 80 total people, the rate per person is reduced by \$2.

It costs \$6000 (a fixed cost) plus \$32 per person to conduct the tour. What number of people will maximize the profit per tour?

6. You are (unjustly) sent to jail. The prison is a tall building surrounded by an 8ft tall fence situated 4ft away from the building. Your buddies are organizing an escape for you. The main tool is a straight ladder (but made from an expensive material). What is the shortest ladder that touches the ground at one end, passes over the fence (touching the fence to deactivate the prison alarm system!) and then touches the building at the other end?

7. James Bond is at the southernmost point of a circular lake with diameter of two miles. He needs to get to the northernmost point (diametrically opposite) of the lake as quickly as possible. It is known that James Bond runs twice as fast as he can swim. He can swim directly across the lake, he can run all the way around the lake, or he can try a combination of swimming and running. What path should James Bond take in order to minimize the time of the trip?