Name: _____ **Panther ID:** _____

Exam 2 Calculus II Fall 2014

To receive credit you MUST SHOW ALL YOUR WORK. Answers which are not supported by work will not be considered.

1. (12 pts) Circle the correct answer (3 pts each):

(a) For the integral $\int \sqrt{4x^2-9}\,dx$, the following substitution is helpful:

(i) $3x = 2\cos\theta$ (ii) $2x = 3\sin\theta$ (iii) $w = 4x^2 - 9$ (iv) $2x = 3\sec\theta$ (v) $2x = 3\tan\theta$

(Don't spend time evaluating the integral. It is not required.)

(b) The partial fraction decomposition of $\dfrac{x+3}{x^4+9x^2}$ is of the form:

(i) $\dfrac{A}{x^2} + \dfrac{B}{x^2+9}$ (ii) $\dfrac{A}{x} + \dfrac{B}{x^2} + \dfrac{C}{x+3} + \dfrac{D}{(x+3)^2}$ (iii) $\dfrac{x+3}{x^4} + \dfrac{x+3}{9x^2}$

(iv) $\dfrac{A}{x} + \dfrac{B}{x^2} + \dfrac{Cx+D}{x^2+9}$ (v) $\dfrac{A}{x} + \dfrac{B}{x^2} + \dfrac{C}{x+3}$

(c) The function $f(x)$ is known to be continuous, positive and concave up when $x \in [-2, 2]$. Let M_4 be the mid-point approximation with 4 subdivisions of the integral $\int_{-2}^{2} f(x)\,dx$. Then compared with the integral, M_4 is an

(i) overestimate (ii) underestimate (iii) exact estimate (iv) cannot tell (more should be known about $f(x)$)

(d) The function $g(x)$ is known to be a quadratic function. Let S_4 be the Simpson approximation with 4 subdivisions of the integral $\int_{-2}^{2} g(x)\,dx$. Then compared with the integral, S_4 is an

(i) overestimate (ii) underestimate (iii) exact estimate (iv) cannot tell (more should be known about $g(x)$)

2. (20 pts) Compute each of the following:

(a) (8 pts) $\displaystyle\int x^3 \ln x\,dx$

(b) (12 pts) $\displaystyle\int \frac{x^3}{\sqrt{4-x^2}}\,dx$

3. (10 pts) The region bounded between $y = \sqrt{x}$ and $y = \frac{1}{2}x$ is rotated around the y-axis. Sketch the solid and set up an integral that represents the volume of this solid. Just set up. The calculation of the integral is not required.

4. (22 pts) Compute each of the following:

(a) (10 pts) $\displaystyle\int_0^1 \arcsin x \, dx$

(b) (12 pts) $\displaystyle\int \frac{x+2}{x(x^2+4)} \, dx$

5. (10 pts) The tank of a fuel truck is a cylinder of radius 3 ft and length 30 ft. The tank sits horizontally with the lower side at an altitude of 2 ft above the ground (wheels of the truck are 2 ft high). Assuming that the tank is initially half-full, set up an integral that represents the total work required to completely fill up the tank by pumping up gasoline from ground level. The density of gasoline is $\rho = 45$ lb/ft^3. (Just set up. The calculation of the integral is not required. Make sure to show on a picture what variable(s) you are using.)

6. (16 pts) Compute each of the following improper integrals. Specify if they are convergent or divergent. Why is the second integral improper?

(a) $\displaystyle\int_0^{+\infty} e^{-3x}\,dx$

(b) $\displaystyle\int_0^2 \frac{1}{(t-1)^2}\,dt$

7. (24 pts) Choose TWO out of the following THREE (12 pts each):

(a) Using the slicing method, prove the formula for the volume of a pyramid. (if needed, you may assume that the base of the pyramid is a square).

(b) Find (with proof) a reduction formula for $\int \tan^n x \, dx$.

(c) Find the formula for surface area of a sphere of radius a, by rotating the semi-circle $x = a\cos t$, $y = a\sin t$, $t \in [0, \pi]$, around the x-axis. Full computation is required.