

Name: _____

Panther ID: _____

FINAL EXAM

Calculus II

Fall 2014

Important Rules:

1. Unless otherwise mentioned, to receive full credit you **MUST SHOW ALL YOUR WORK**. Answers which are not supported by work might receive no credit.
2. Please turn your cell phone off at the beginning of the exam and place it in your bag, **NOT** in your pocket.
3. No electronic devices (cell phones, calculators of any kind, etc.) should be used at any time during the examination. Notes, texts or formula sheets should **NOT** be used either. Concentrate on your own exam. Do not look at your neighbor's paper or try to communicate with your neighbor.
4. Solutions should be concise and clearly written. Incomprehensible work is worthless.

1. (20 pts) In each case circle True or False. No justification needed. (2 pts each)

(a) Every continuous functions has an anti-derivative. **True False**

(b) Every bounded sequence is convergent. **True False**

(c) If f is positive and concave up on $[a, b]$, then $M_n \leq \int_a^b f(x)dx$. **True False**

(d) If f is strictly increasing on $[a, b]$, then $L_n < T_n < R_n$. **True False**

(e) The average value of f on $[a, b]$ is $\frac{f(a)+f(b)}{2}$. **True False**

(f) If $0 < a_k < \frac{1}{k}$ for all $k \geq 1$, then the series $\sum_{k=1}^{\infty} a_k$ is convergent. **True False**

(g) If $0 < a_k < \frac{1}{k}$ for all $k \geq 1$, then the sequence $\{a_k\}$ is convergent. **True False**

(h) $\int_0^{+\infty} \cos x = 0$. **True False**

(i) Distance traveled can be less than displacement. **True False**

(j) The integral $\int_1^5 \frac{1}{(x-3)^2} dx$ is improper. **True False**

2. (20 pts) Evaluate

(a) $\int_0^{\pi/4} \sqrt{\tan x} \sec^2 x dx$

(b) $\int x^2 \ln x dx$

3. (20 pts) Evaluate

(a) $\int \frac{1}{x(2x+1)} dx$

(b) $\int \frac{1}{(25-x^2)^{3/2}} dx$

4. Choose ONE. Note the different point values.

(a) (15 pts) State and prove the Geometric Series Theorem.

(b) (10 pts) State and prove the integration by parts formula.

5. (30 pts) Sketch a picture and then set up an integral representing each of the following. Computation of the integral is **not** required for this problem.

(a) Volume of the solid obtained by revolving the region bounded by $x = 0$, $2x + y = 4$ and $y = 0$ around the line $x = -1$ (sketch required).

(b) Area of the region inside the cardioid $r = 1 + \sin \theta$ and below the x -axis. Sketch required.

(c) Work. A vertical cylindrical tank of radius 5 ft and height 9 ft is two-thirds filled with gasoline of density $\rho = 50 \text{ lbs/ft}^3$. Set up an integral for the work required to pump all the gasoline over the upper rim.

6. (24 pts) Determine whether each of the following series is absolutely convergent (AC), conditionally convergent (CC), or divergent. Be sure to state which test you are using and to show that it applies to the series in question.

(a)
$$\sum_{k=1}^{+\infty} (-1)^{k-1} \frac{1}{k^2 + 1}$$

(b)
$$\sum_{k=2}^{+\infty} (-1)^k \frac{\ln k}{k}$$

8. (20 pts) (a) (12 pts) Find the interval of convergence (with endpoints) of the series $\sum_{k=1}^{\infty} \frac{(-1)^{k-1}}{k \cdot 3^{k-1}} (x-5)^k$.

(b) (8 pts) Determine a function $f(x)$ whose Taylor series at $x_0 = 5$ is the series in part (a). Hint: Find first $f'(x)$.

9. (16 pts) (a) (10 pts) The function $f(x) = \frac{1}{2}(e^x + e^{-x})$ is called the hyperbolic cosine of x , and is traditionally denoted $\cosh x$. Find its MacLaurin series. Your final answer should use summation notation.

(b) (6 pts) Based on part (a), one writes the approximation $\cosh(0.2) \approx 1 + \frac{(0.2)^2}{2}$. Can you guarantee that the error in this approximation is less than 0.001? Explain your answer.

Hint: Recall that $|R_n(x)| \leq \frac{M}{(n+1)!}|x - x_0|^{n+1}$ and also note that, for $\cosh x$, the MacLaurin polynomials of degrees 2 and 3 coincide.