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## Worksheet week 1 <br> - MAC 2312, Fall 2014

1. ( 6 pts ) Determine a formula for the general term $a_{n}$ of the sequence and decide whether the sequence converges or diverges. Briefly justify each case.
(a) $a_{1}=1, a_{2}=-1, a_{3}=1, a_{4}=-1, \ldots$
(b) $a_{0}=1, a_{1}=-\frac{2}{3}, a_{2}=\frac{4}{9}, a_{3}=-\frac{8}{27}, \ldots$
(c) $a_{1}=1, a_{2}=\frac{1}{2}, a_{3}=1, a_{4}=\frac{2}{3}, a_{5}=1, a_{6}=\frac{3}{4}, \ldots$
2. ( 6 pts ) In this problem you will prove (with Calculus) that the area of a circle of radius $r$ is given by $A=\pi r^{2}$.
(a) Consider a regular pentagon inscribed in this circle and let $A_{5}$ denote the area of this pentagon. Find a formula for $A_{5}$ in terms of the radius $r$ of the circle (of course, some factor involving $\sin (\pi / 5)$ will also appear).
Hint: Using the center of the circle, divide the pentagon into 5 congruent triangles.
(b) Consider now a regular polygon with $n$-sides inscribed in this circle and let $A_{n}$ denote the area of this pentagon. Following your reasoning in part (a), find a formula for $A_{n}$ in terms of the radius $r$ of the circle.
(c) Give an informal reason why $A=\lim _{n \rightarrow+\infty} A_{n}$ and then compute the limit to get the famous $A=\pi r^{2}$.
