Name:

Worksheet week 7 Calculus II Fall 2014

1. The region bounded between the graph of $\sin x$ and the x-axis when $x \in [0, \pi]$ is rotated around the y-axis; the solid formed has volume V_1 . Then the same region is rotated around the x-axis; the solid formed has volume V_2 . Find V_1 and V_2 and observe that $V_1 = 4V_2$.

2. Evaluate (a)
$$\int \sin^2 x \cos^3 x \, dx$$
 (b) $\int \tan^2 x \sec^4 x \, dx$

3. (a) Derive a reduction formula for

$$\int \sin^n x \ dx \ ,$$

where n is a positive integer. You may check formula (9) in 7.2 to confirm your result.

(b) Use part (a) to derive a recursion formula for

$$A_n = \int_0^{\pi/2} \sin^n x \, dx \, .$$

(c) Find A_1 directly, then find A_3 , A_5 using the recursion formula. Write a general formula for A_n when n is odd. (d) Find A_0 directly, then find A_2 , A_4 using the recursion formula. Write a general formula for A_n when n is even. The general formulas for A_n are the so-called *Wallis sine formulas*.