NAME: Solution Key Panther ID: ____________

Worksheet week 8 - MAC 2312, Spring 2014

1. Write out the form of the partial fraction decomposition for $\frac{x^2+x+1}{(x^2+1)(x+1)^2}$. You do not have to determine the numerical value of the coefficients.

$$\frac{x^2+x+1}{(x^2+1)(x+1)^2} = \frac{A}{(x+1)} + \frac{B}{(x+1)^2} + \frac{Cx+D}{x^2+1}$$

2. Compute

(a) $\displaystyle\int \frac{1}{x^4+x^2}\,dx$ (b) $\displaystyle\int \frac{1}{(x+a)(x+b)}\,dx$

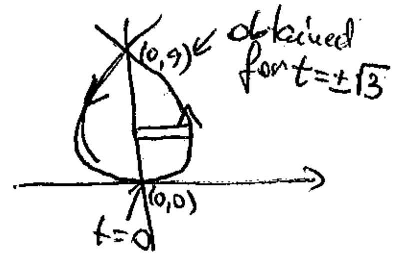
Note: For both integrals, the partial fraction method is the most natural. However, the first integral can also be done nicely with a trigonometric substitution.

See next page

3. You will be shown the graph of the parametric curve $x = 3t - t^3$, $y = 3t^2$.

(a) Find the length of the loop described by the curve.

(b) Write an integral that will compute the area inside of the loop.



(a) $s = \int_?^? ds = \int_?^? \sqrt{(dx)^2+(dy)^2}$

Since the curve is a parametric curve

$dx = x'(t)dt$ $dy = y'(t)dt$

So $$s = \int_{t=-\sqrt{3}}^{t=\sqrt{3}} \sqrt{(x'(t))^2+(y'(t))^2}\,dt \underset{\text{symmetry}}{=} 2\int_0^{\sqrt{3}} \sqrt{(3-3t^2)^2+(6t)^2}\,dt$$

$$s = 2\int_0^{\sqrt{3}} \sqrt{9-18t^2+9t^4+36t^2}\,dt = 2\int_0^{\sqrt{3}} \sqrt{9+18t^2+9t^4}\,dt = 2\int_0^{\sqrt{3}} \sqrt{(3+3t^2)^2}\,dt$$

$$s = 2\int_0^{\sqrt{3}} (3+3t^2)\,dt = 2(3t+t^3)\Big|_0^{\sqrt{3}} = 2\left(3\sqrt{3}+(\sqrt{3})^3\right) = \boxed{12\sqrt{3}}$$

(b) By symmetry, $A = 2\cdot(\text{area in } 1^{st} \text{ quadrant}) = 2\cdot A_1$

Use horizontal strips, $A_1 = \int_?^? l_{strip}\cdot Th_{strip}$

$Th_{strip} = dy = y'(t)dt$

$l_{strip} = x = x(t)$

$$A_1 = \int_{t=0}^{t=\sqrt{3}} x(t)y'(t)\,dt = \int_0^{\sqrt{3}} (3t-t^3)\cdot 6t\,dt = \int_0^{\sqrt{3}} (18t^2-6t^4)\,dt$$

$$A_1 = \left(6t^3 - \frac{6t^5}{5}\right)\Big|_0^{\sqrt{3}} = 6(\sqrt{3})^3\left(1-\frac{3}{5}\right) = \boxed{\frac{36\sqrt{3}}{5}}$$

Thus $A = 2A_1 = \boxed{\frac{72\sqrt{3}}{5}}$

Solution of Pb. 2 - Worksheet 8

(a) $\int \frac{1}{x^4+x^2}\,dx = \int \frac{1}{x^2(x^2+1)}\,dx$

It is easy (in this case) to guess the partial fraction decomposition.

Observe that $(x^2+1) - x^2 = 1$

Thus $\frac{1}{x^2(x^2+1)} = \frac{(x^2+1)-x^2}{x^2(x^2+1)} = \frac{\cancel{x^2+1}}{x^2\cancel{(x^2+1)}} - \frac{\cancel{x^2}}{\cancel{x^2}(x^2+1)} = \frac{1}{x^2} - \frac{1}{x^2+1}$

Thus $\int \frac{1}{x^2(x^2+1)}\,dx = \int \left(\frac{1}{x^2} - \frac{1}{x^2+1}\right)dx = -\frac{1}{x} - \arctan x + c$

Solution with trig. subst. ~~sketch with~~

$x = \tan\theta \Rightarrow x^2+1 = \tan^2\theta + 1 = \sec^2\theta$

$dx = \sec^2\theta\,d\theta$

$\int \frac{1}{x^2(x^2+1)}\,dx = \int \frac{\cancel{\sec^2\theta}\,d\theta}{\tan^2\theta\,\cancel{\sec^2\theta}} = \int \cot^2\theta\,d\theta = \int (\csc^2\theta - 1)\,d\theta =$

$= -\cot\theta - \theta + c = -\frac{1}{x} - \arctan x + c$

$\cot\theta = \frac{1}{\tan\theta} = \frac{1}{x}$

(b) $\frac{1}{(x+a)(x+b)} = \frac{A}{x+a} + \frac{B}{x+b} \quad \Big| \cdot (x+a)(x+b) \Rightarrow$

$1 = A(x+b) + B(x+a)$. Taking $x=-a \Rightarrow 1 = A(-a+b) \Rightarrow A = \frac{1}{b-a}$

Taking $x=-b \Rightarrow 1 = B(-b+a) \Rightarrow B = \frac{1}{a-b} = -\frac{1}{b-a}$

Thus $\int \frac{1}{(x+a)(x+b)}\,dx = \int \frac{1}{b-a}\left(\frac{1}{x+a} - \frac{1}{x+b}\right)dx =$

$= \frac{1}{b-a}\left(\ln|x+a| - \ln|x+b|\right) = \boxed{\frac{1}{b-a}\ln\left(\frac{|x+a|}{|x+b|}\right) + c}$