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Quiz 1 - MAC 2312, Spring 2016

1. True/False questions. In each case, circle your answer (1 pt) and briefly justify (2 pts).

(a) (3 pts) The sequence $a_n = n^2 10^{-n}$, $n \geq 1$ is monotone. **True** **False**

Justification:

$$\frac{a_{n+1}}{a_n} = \frac{(n+1)^2}{10^{n+1}} \cdot \frac{10^n}{n^2} = \frac{(n+1)^2}{10n^2} < 1 \text{ for all } n \geq 1$$

(since $(n+1)^2 < 10n^2 \Leftrightarrow$
$\Leftrightarrow n^2+2n+1 < 10n^2 \Leftrightarrow$
$\Leftrightarrow 2n+1 < 9n^2 \ \forall n \geq 1$,
obviously true)

(b) (3 pts) Any bounded sequence is convergent. **True** **False**

Justification:

$a_n = (-1)^n$ is bounded but is divergent

(Of course, many other examples are possible.)

2. (a) (3 pts) Determine if the series $\sum_{k=2}^{\infty}\left(\frac{1}{k-1}-\frac{1}{k+1}\right)$ converges and if so find its limit.

Observe the series is telescopic.

$$S_n = \sum_{k=2}^{n}\left(\frac{1}{k-1}-\frac{1}{k+1}\right) = \left(\frac{1}{1}-\cancel{\frac{1}{3}}\right)+\left(\frac{1}{2}-\cancel{\frac{1}{4}}\right)+\left(\cancel{\frac{1}{3}}-\cancel{\frac{1}{5}}\right)+\left(\cancel{\frac{1}{4}}-\cancel{\frac{1}{6}}\right)+\dots$$

$$\dots+\left(\cancel{\frac{1}{n-2}}-\frac{1}{n}\right)+\left(\cancel{\frac{1}{n-1}}-\frac{1}{n+1}\right)$$

Thus $S_n = \frac{1}{1}+\frac{1}{2}-\frac{1}{n}-\frac{1}{n+1}$, so $\lim_{n\to+\infty} S_n = 1+\frac{1}{2} = \frac{3}{2}$

So $\sum_{k=2}^{\infty}\left(\frac{1}{k-1}-\frac{1}{k+1}\right)$ converges to $\frac{3}{2}$.

(b) (3 pts) Determine if the series $\frac{1}{3}-\frac{2}{9}+\frac{4}{27}-\frac{8}{81}+\frac{16}{243}-\dots$ converges and if so find its limit.

$$= \sum_{k=0}^{\infty}\frac{(-1)^k \cdot 2^k}{3^{k+1}} = \frac{1}{3}\sum_{k=0}^{\infty}\frac{(-1)^k 2^k}{3^k} = \frac{1}{3}\sum_{k=0}^{\infty}\left(-\frac{2}{3}\right)^k$$

geometric series with $r = -\frac{2}{3}$, so $|r| = \frac{2}{3} < 1$
so the series converges.

Its limit is $\frac{1}{3} \cdot \frac{1}{1-\left(-\frac{2}{3}\right)} = \frac{1}{3} \cdot \frac{3}{5} = \boxed{\frac{1}{5}}$