NAME: ______________________________ **Panther ID:** ____________

Worksheet week 2 - MAC 2312, Spring 2016

1. (a) Find a simple closed form for the sum $\frac{1}{2^2-1}+\frac{1}{3^2-1}+\frac{1}{4^2-1}+ \ldots +\frac{1}{n^2-1}$

Hint: Check that $\frac{1}{k^2-1} = \frac{1}{(k-1)(k+1)} = \frac{1}{2}\left(\frac{1}{k-1} - \frac{1}{k+1}\right)$ and note that you get a telescopic sum.

(b) Use the result in part (a) to find

$$\lim_{n\to+\infty} \sum_{k=2}^{n} \frac{1}{k^2-1}$$

Note: This limit is, by definition, the series $\sum_{k=2}^{+\infty} \frac{1}{k^2-1}$.

Thus, you proved that the series above is convergent and you found its exact sum.

2. Applications of geometric series theorem.

(a) Find the sum of the series (if it exists) $2/3 + 4/9 + 8/27 + 16/81 + \ldots$

(b) Find the sum of the series (if it exists) $\sum_{k=2}^{+\infty} \frac{(-3)^k}{2^{2k+1}}$

(c) Express the number 0.37777777... as a ratio of two integers.

(d) Express the number 0.13131313... as a ratio of two integers.

Note: Generalizing the ideas from (c) and (d), one can prove that any periodic number (that is, any number whose decimal digits repeat) is, in fact, a rational number.