Worksheet week 7 - MAC 2312, Spring 2016 **NAME:** Solution Key

1. Evaluate (a) $\int \sin^2 x \cos^3 x \, dx =$

$$= \int \sin^2 x \cos^2 x \cos x \, dx =$$

$$= \int \sin^2 x (1 - \sin^2 x) \cos x \, dx =$$

sub $w = \sin x$

$dw = \cos x \, dx$

$$= \int w^2 (1 - w^2) \, dw = \int (w^2 - w^4) \, dw$$

$$= \frac{w^3}{3} - \frac{w^5}{5} + c = \boxed{\frac{\sin^3 x}{3} - \frac{\sin^5 x}{5} + c}$$

(b) $\int \tan^2 x \sec^4 x \, dx =$

$$= \int \tan^2 x \sec^2 x \sec^2 x \, dx =$$

$$= \int \tan^2 x (\tan^2 x + 1) \sec^2 x \, dx$$

sub $w = \tan x$

$dw = \sec^2 x \, dx$

$$= \int w^2 (w^2 + 1) \, dw = \int (w^4 + w^2) \, dw$$

$$= \frac{w^5}{5} + \frac{w^3}{3} + c =$$

$$\boxed{= \frac{\tan^5 x}{5} + \frac{\tan^3 x}{3} + c}$$

(c) $\int \frac{1}{3x^2 + 4} \, dx = \int \frac{1}{\left(\frac{3x^2}{4} + 1\right) \cdot 4}$

$$= \frac{1}{4} \int \frac{1}{\left(\frac{\sqrt{3} x}{2}\right)^2 + 1} \, dx =$$

sub $w = \frac{\sqrt{3}}{2} \cdot x$

$dw = \frac{\sqrt{3}}{2} \, dx$

$$= \frac{1}{4} \int \frac{\frac{2}{\sqrt{3}} \, dw}{w^2 + 1} =$$

$$= \frac{1}{4} \cdot \frac{2}{\sqrt{3}} \arctan w + c =$$

$$\boxed{= \frac{1}{2\sqrt{2}} \arctan\left(\frac{\sqrt{3}}{2} x\right) + c}$$

(d) $\int \frac{x}{\sqrt{1 - x^4}} \, dx =$

$$= \int \frac{x}{\sqrt{1 - (x^2)^2}} \, dx =$$

sub. $w = x^2$

$dw = 2x \, dx \Rightarrow \frac{1}{2} dw = x \, dx$

$$= \int \frac{\frac{1}{2} dw}{\sqrt{1 - w^2}} = \frac{1}{2} \arcsin w + c$$

$$\boxed{= \frac{1}{2} \arcsin(x^2) + c}$$

2. The tank of a fuel truck is a cylinder of radius 3 ft and length 30 ft. The tank sits horizontally with the lower side at an altitude of 2 ft above the ground (wheels of the truck are 2 ft high). Assuming that the tank is initially half-full, set up an integral that represents the total work required to completely fill up the tank by pumping up gasoline from ground level. The density of gasoline is $\rho = 45$ lb/ft^3. (Just set up. The calculation of the integral is not required. Make sure to show on a picture what variable(s) you are using.)

Solved in class. See your notes

3. Suppose you have to drill a narrow but deep pit into the ground. The pit is cylindrical, with a radius of 1ft and with a depth of 1000ft. The density of the rock encountered varies, so assume that at a depth of x ft from the ground, the density is given by some function $\rho(x)$ lbs/ft^3.

(a) Write a formula to express the total mass of the material removed during drilling.

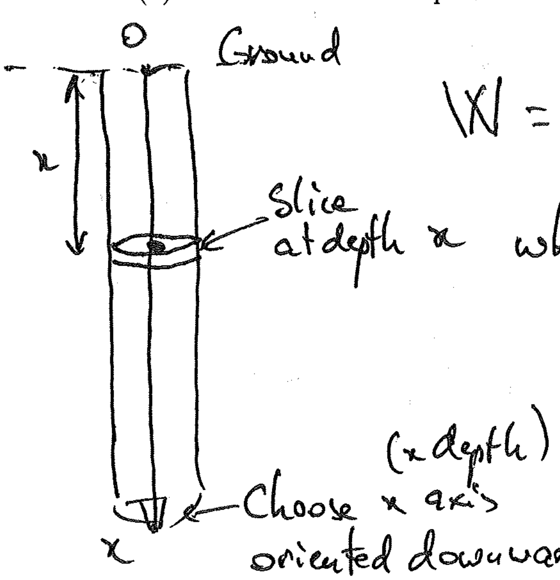
Since the density varies with depth, we should slice the pit with thin horizontal slices.

$$\text{mass} = m \approx \sum m_{slice} \approx \sum \rho(x) \cdot A_{slice} \cdot Th_{slice} \xRightarrow{\text{Take limit}} m = \int_{x=0}^{x=1000} \rho(x) \cdot \pi \cdot 1^2 \, dx$$

so

$$\boxed{m = \pi \int_0^{1000} \rho(x) \, dx}$$ Answer for (a)

(b) Write a formula to express the total work done in removing the drilled material to the ground level.



$$W = \int_?^? dW$$

where dW = work to get the slice at depth x to the surface

$$dW = \text{weight}_{slice} \cdot \text{dist}_{slice}$$

$$dW = \rho(x) \cdot V_{slice} \cdot \text{dist}_{slice} = \rho(x) \cdot A_{slice} \cdot \text{dist}_{slice} \cdot Th_{slice}$$

but $A_{slice} = \pi \cdot 1^2 = \pi$ so

$$W = \int_{x=0}^{x=1000} \rho(x) \cdot \pi \cdot x \, dx = \boxed{\pi \int_0^{1000} x \rho(x) \, dx}$$