

To receive credit you MUST SHOW ALL YOUR WORK.

1. (2 pts) If $h'(t)$ is the rate of change of a child's height measured in inches per year, what does the integral $\int_2^8 h'(t) dt$ represent and what are its units?

By FTC, $\int_2^8 h'(t) dt = h(8) - h(2)$, so this represents the change in height of the child from age 2 to age 8. Units \rightarrow inches

2. (3 pts) Is the statement below true or false? Answer (1 pt) and briefly justify your answer (2 pts).

There does not exist a differentiable function $F(x)$ such that $F'(x) = |x|$. True False

Justification: By F.T.C. part (b) ^{since $|x|$ is continuous} the function

$$F(x) = \int_0^x |t| dt \text{ satisfies } F'(x) = |x|$$

(or, shorter said, any continuous function has an anti-derivative! and $y = |x|$ is continuous)

3. (6 pts) Evaluate each integral (3 pts each):

(a) $\int_1^4 \frac{1}{x\sqrt{x}} dx =$

$$= \int_1^4 x^{-\frac{3}{2}} dx =$$

$$= (-2)x^{-\frac{1}{2}} \Big|_{x=1}^{x=4}$$

$$= -\frac{2}{\sqrt{x}} \Big|_{x=1}^{x=4}$$

$$= -\frac{2}{\sqrt{4}} + \frac{2}{\sqrt{1}} = \boxed{1}$$

(b) $\int_0^{\frac{5\pi}{6}} |\cos x| dx$

$$= \int_0^{\frac{\pi}{2}} |\cos x| dx + \int_{\frac{\pi}{2}}^{\frac{5\pi}{6}} |\cos x| dx =$$

(in 1st quadrant $\cos x > 0$) (in 2nd quadrant $\cos x < 0$)

$$= \int_0^{\frac{\pi}{2}} \cos x dx + \int_{\frac{\pi}{2}}^{\frac{5\pi}{6}} (-\cos x) dx$$

$$= (\sin x) \Big|_{x=0}^{x=\frac{\pi}{2}} - (\sin x) \Big|_{x=\frac{\pi}{2}}^{x=\frac{5\pi}{6}}$$

$$= \sin \frac{\pi}{2} - \sin 0 - \sin \frac{5\pi}{6} + \sin \frac{\pi}{2}$$

$$= 1 - 0 - \frac{1}{2} + 1 = \boxed{\frac{3}{2}}$$