Worksheet week 11 - MAC 2312, Spring 2017 **NAME:** ____________________

1. (similar to Pb. 31 in section 7.7) (a) Show that the exact value of the integral below is π.

$$\int_0^2 \frac{8}{x^2+4}\,dx$$

(b) Compute the approximations M_8, T_8 and S_{16} for the integral above.

2. The goal of this exercise is to give another proof of the fact that the harmonic series is divergent. The same idea is used to prove the integral test for series.

(a) Let $n \geq 2$ be an integer.
Find the area below the graph of $f(x) = 1/x$
and above the x-axis on the interval $[1, n]$.
(your answer will, of course, depend on n).

(b) Shade the left endpoint Riemann sum
approximation of the area in part (a)
for the division of the interval $[1, n]$
into sub-intervals of length 1.
Write the expression for this Riemann sum.

(c) Explain why from parts (a) and (b) you obtain the inequality

$$\frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \ ... \ + \frac{1}{n-1} \geq \ln n \ , \text{ and further, from this, } \sum_{k=1}^{+\infty} \frac{1}{k} = +\infty \ .$$