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Worksheet week 2 - MAC 2312, Spring 2017

1. (The basketball example revisited) A basketball tossed straight up in the air (from the ground) reaches a high-point  $h_0$  and falls to the floor. Each time the ball bounces to the floor it rebounds to a (constant) ratio  $r$  of its previous height ( $0 < r < 1$ ). Assume the initial height is  $h_0 = 20$  ft, and the ratio  $r = 3/4$ . Let  $h_k$  be the high-point of the ball after the  $k$ -th bounce.

(a) In the previous worksheet you found the general formula for  $h_k$ . Write it again now.

(b) In principle, the ball will do infinitely many bounces (although we will not be able to perceive them at some point). Accepting this, do you think the total distance travelled by the ball is infinite? Just answer Yes or No, you'll have another opportunity to reflect on your answer.

(c) Set up a series that describes the total distance travelled by ball and find the sum of the series.

(d) What is the conclusion of the result in (c)? Was your answer in part (b) correct?

2. This problem refers again to the previous example, but this time we are interested in the total amount of time the ball is bouncing.

(a) Assuming again that the ball is doing infinitely many bounces, do you think that the total time that the ball is bouncing is infinite? Again, just answer Yes or No, you'll reflect on your answer later.

(b) It is known (and can be proved with calculus) that if an object is thrown straight up in the air from the ground and it reaches a maximum height  $\tilde{h}$  feet, then the time it takes to reach this height is  $\tilde{t} = \sqrt{2\tilde{h}/g}$ , where  $g$  is the gravitational acceleration 32 ft/s<sup>2</sup>. (This is true neglecting air resistance, which we also assume in our example.) Using this fact, set up a series that gives the total time travelled by the ball in the previous problem. You can again assume that  $h_0 = 20$  and  $r = 3/4$ .

(c) Find the sum of the series. Was your answer to part (a) correct?

3. (Home reading) You might have heard about Zeno's paradox of Achilles and the Tortoise (that Achilles, the fastest of all mortals, can never catch up with the Tortoise). Do an internet search and read about this paradox and its relation with series. Problems 1 and 2 are very much in the same spirit.