Worksheet week 4 - MAC 2312, Spring 2017 **NAME:** ____________________

1. Use substitution to compute each integral:

(a) $\displaystyle\int_0^1 xe^{-x^2}\,dx$

(b) $\displaystyle\int_0^1 \frac{x^2}{2x^3+1}\,dx$

(c) $\displaystyle\int_e^{e^2} \frac{1}{x\sqrt{\ln x}}\,dx$

(d) $\displaystyle\int_0^{\pi/2} \frac{\sin(2x)}{2+\cos(2x)}\,dx$

2. Show that the Fresnel sine function $S(x) = \int_0^x \sin(t^2)\,dt$ satisfies the differential equation

$(S'(x))^2 + \left(\frac{S''(x)}{2x}\right)^2 = 1$.

3. Given that $F(x) = \displaystyle\int_0^x \sqrt{8t - t^2}\,dt$, for $x \in [0, 8]$, do the following:

(a) Determine the values of $F(0)$, $F(4)$, $F(8)$. Hint: Complete the square and use geometry.
(b) Determine $F'(x)$ and $F''(x)$.
(c) Based on parts (a) and (b), sketch the graph of the function $y = F(x)$, for $x \in [0, 8]$. What kind of point is $x = 4$ for the graph of $y = F(x)$?