Name: _____ **Panther ID:** _____

Take home part of the Final Exam MAT 3501 Fall 2017

This is the take home part of the final exam, due Tuesday, Dec. 12. You are encouraged to collaborate, ask questions, but each of you should understand the solutions that you are handing in. You should also acknowledge collaborations or outside help.

1. For each of the following, answer if the statement is True or False. Then give a brief justification of your answer.

(a) For any prime number p, $p(p+1)+1$ is also prime. **True** **False**

Justification:

(b) There exist infinitely many prime numbers p, so that $p(p+1)+1$ is not prime. **True** **False**

Justification:

(c) If p is a prime number, $\sqrt[k]{p}$ is irrational, for any $k \geq 2$ integer. **True** **False**

Justification:

(d) If p is a prime number, $p^{\frac{1}{p}}$ is transcendental. **True** **False**

Justification:

(e) For any natural number n, $n^{n^{1/n}} = n$. **True** **False**

Justification:

(f) If p is a prime number, $p^{p^{1/p}}$ is transcendental. **True** **False**

Justification:

(g) Suppose $q(x)$ is a polynomial with integer coefficients and leading coefficient 1. If $q(0) = p$, where p is a prime number and $q(1) = 1$, then $q(x)$ does not have any rational root. **True** **False**

Justification:

2. Suppose there is a rail-road track along the graph of $f(x) = 2^x$. Lets call it the 2^x-roadway to infinity! Suppose you live in a city located at the origin $(0, 0)$, so your city is not on the track to infinity. You want to remedy this situation, so you plan to built a railroad from your city to connect smoothly with the 2^x-roadway to infinity. The problem is that your city can only produce straight railroad track. Can you build a straight railroad so that you take a train from your city at $(0, 0)$, get to some point on $y = 2^x$ and from there the train shifts smoothly onto the 2^x-track to infinity? Be careful, you dont want the train to derail at the connection!

3. Water is stored in a cone-shaped reservoir (vertex down) open at the top. Assuming the water evaporates at a rate proportional to the surface area exposed to the air, show that the depth of the water will decrease at a constant rate that does not depend on the dimensions of the reservoir.

4. James Bond is at the southernmost point of a circular lake. He needs to get to the northernmost point (diametrally opposite). It is known that James Bond runs twice as fast as he can swim and he can maintain his top speed (both for running and for swimming) for a long time. He can swim directly across the lake, he can run all the way around the lake, or he can try a combination of swimming and running.

(a) What path should James Bond take in order to minimize the time of the trip?

(b) What path should James Bond take in order to maximize the time of the trip (but keeping the appearance that he is doing his best)?

(c) Create a James Bond story (or Mr. Bean James Bond story) that would fit the scenario in part (b).

5. (a) How many times in 24 hours are the two arms of a clock perpendicular to each other? Assume continuous motion for both arms.

(b) Find the exact times between 4pm and 5pm, when the two arms of a clock are perpendicular.

(c) Consider now a clock with three arms (one for hours, one for minutes, and one for seconds) with continuous motion for all three arms. Find, if possible, a moment when the hours and the minutes arms form an 180° angle and the seconds arm is perpendicular to both the other arms.