**Name:** ____________________________ **Panther ID:** ______________

**Worksheet - Aug. 29** MAT 3501 Fall 2017

**1.** In each case, prove using induction:

(a) For all $n \geq 0$, $3^{2n+1} + 2^{n+2}$ is divisible by 7.

(b) (pb. 14 section 8.3) Show that the sum of the interior angles of a convex polygon is $180(n-2)$ degrees, where $n$ is the number of sides of the polygon.

(c) For all $n \geq 1$, $1^2 + 2^2 + 3^2 + \ \dots \ + n^2 = \frac{n(n+1)(2n+1)}{6}$.

(d) Show that $2^n | (n+1)(n+2) \dots (2n)$, for all positive integers $n$.

**Definitions:** (i) The **greatest common divisor** of a set of numbers is the largest of the divisors common to all the numbers. (*Notation:* gcd(...).)

(ii) The **least common multiple** of a set of numbers is the smallest of the multiples common to all the numbers. (*Notation:* lcm(...).) Note that this definition uses the Well Ordering Principle!

(iii) Two numbers $a, b$ are relatively prime if they share no prime common factor, that is, if $\gcd(a, b) = 1$.

**2.** Use the Division Theorem and the Well Ordering Principle to prove the following important result:
**Theorem:** If $a, b$ are positive integers, show that there exist integers $x, y$ so that $ax + by = gcd(a, b)$.
*Start of Proof:* Consider the set $S = \{ax + by > 0 | x, y \in \mathbf{Z}\}$. By well ordering principle, the set $S$ has a smallest element, call it $d$. Show that $d = gcd(a, b)$.

**Corollary:** If $a, b$ are relatively prime, i.e. if $\gcd(a, b) = 1$, then there exist integers $x, y$ so that $ax + by = 1$.