

Name: _____

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Homework 1 - Topology – Fall 2015

Due Thursday, Sept. 3, 2015

1. (version of Pb. 11, page 15 in the text – exercises for section 1.2) On the set $\mathcal{X} = \{a, b, c, d\}$ find two topologies $\mathcal{T}_1, \mathcal{T}_2$ so that $\mathcal{T}_1 \cup \mathcal{T}_2$ is not a topology on \mathcal{X} .

2. Exercise 6, page 25 in the text (section 1.3).

3. Exercise 16, page 9 in the text (section 1.1).

4. Suppose that d and ρ are two metrics defined on the same set X . We say that d and ρ are *equivalent* if they induce the same topology on X . We say that d and ρ are *strongly equivalent* if there exist positive constants A, B so that

$$A\rho(x, y) \leq d(x, y) \leq B\rho(x, y) \text{ for all } x, y \in X.$$

(a) Prove that if d and ρ are strongly equivalent then they are equivalent.

(b) Show that the Euclidean metric and the square metric on \mathbf{R}^n are strongly equivalent.

(c) Show that the Euclidean metric and the taxi-cab metric on \mathbf{R}^n are strongly equivalent.

(d) Give an example to show that the converse of the statement in part (a) is not true. *Hint:* You may want to look at Exercise 3, page 42, section 1.5 textbook. No new material beyond of what you know already is needed to solve it.