Name: ________________________ **PantherID:** ________________________

Homework 2 - Topology – Fall 2015 **Due Thursday, Sept. 17, 2015**

1. Exercise 15, page 35 in the text (section 1.4).

2. Exercise 28, page 37 in the text (section 1.4).

Clarification of definitions:

A point x of a topological space X is called *an isolated point of* X if the singleton set $\{x\}$ is open in X.

If X is a topological space and A is a subset of X, a point $a \in A$ is said to be *an isolated point of* A if there exists an open set U in X such that $U \cap A = \{a\}$.

If X is a topological space and A is a subset of X, then A is called *relatively discrete* if all its points are isolated points (of A).

3. Exercise 13, page 42-43 in the text (section 1.5).

Note: For definitions of a G_δ-set and F_σ-set see the bottom of page 41 in the text.

4. (a) Prove that in a Hausdorff space every singleton $\{x\}$ is a closed set.

(b) Show that $\mathbb{R}$ with the finite complement topology is not a Hausdorff space, but every singleton $\{x\}$ is a closed set in $\mathbb{R}$ with the finite complement topology.

(c) Show that if $\{x_n\}_n$ is a sequence in $\mathbb{R}$ with $x_n \neq x_m$ for $n \neq m$, then, with respect to the finite complement topology on $\mathbb{R}$, *for every* $y \in \mathbb{R}$, $\{x_n\}_n$ converges to y.

Note: Without the assumption $x_n \neq x_m$ for $n \neq m$, the above statement is not true. There are sequences which converge to a unique limit even in the finite complement topology. Can you give an example? (bonus 1 pt)

(d) Can you find an example of a topological space where every sequence converges to every point in the space? (Hint: This exercise is *trivial*.)