Name: ______________________________ **PantherID:** ____________________

Midterm - Topology – Fall 2015

1. (20 pts) Define each of the following:

(i) A metric on a set X

(ii) The set of limit points A' of a set A in a topological space $(X, \mathcal{T})$

(iii) A local basis at a point x in a topological space $(X, \mathcal{T})$

(iv) A second countable topological space

(v) A homeomorphism

2. (6 pts) Give an example of a topological space $(X, \mathcal{T})$, a subspace $(A, \mathcal{T}_A)$ of $(X, \mathcal{T})$, and a closed set in $(A, \mathcal{T}_A)$ that is not closed in $(X, \mathcal{T})$.

3. (12 pts) Let $(X, \mathcal{T})$ be a topological space and let $f, g : X \to \mathbb{R}$ be continuous functions. Show that the function $h : X \to \mathbb{R}$ defined by $h(x) = f(x) + g(x)$ is continuous.

4. (20 pts) Let $X = [0, +\infty)$ be the set of non-negative real numbers and let $\mathcal{T} \subseteq \mathcal{P}(X)$ be defined by

$$\mathcal{T} := \{U \in \mathcal{P}(X) \mid U = \emptyset, \text{ or } U = X, \text{ or } U = (a, +\infty), \text{ for some } a > 0\}$$

(a) (8 pts) Show that $\mathcal{T}$ is a topology on X.

(b) (6 pts) Consider the sequence $\{x_n\}_{n=1}^{\infty}$ in X, where $x_n = n$, for every positive integer n. Is the sequence $\{x_n\}_{n=1}^{\infty}$ convergent in $(X, \mathcal{T})$? Justify your answer.

(c) (6 pts) Is $(X, \mathcal{T})$ a metrizable topological space? Justify your answer.

5. (14 pts) Let $A = (-1, 1) = \{x \in \mathbb{R} \mid -1 < x < 1\}$. Find the closure, the interior and boundary of A in $\mathbb{R}$ with the lower limit topology. Justify your answers.

6. (12 pts) Consider $\mathbb{R}^\omega$, the countable infinite product of $\mathbb{R}$ with itself.

Define the function $f : \mathbb{R} \to \mathbb{R}^\omega$, by $f(t) = (t, t, t, \ ...)$.

Show that if $\mathbb{R}^\omega$ is given the box topology then f is not continuous.

Choose TWO of the following THREE problems on this page.

7. (12 pts) Let (X, d) be a complete metric space. Show that a subspace A is complete (with the induced metric) if and only if A is closed in X.

8. (12 pts) Let A be an open subset of a separable space $(X, \mathcal{T})$. Prove that $(A, \mathcal{T}_A)$ is separable.

9. (12 pts) Let $(X, \mathcal{T})$ be topological a space and let $\mathcal{U}$ denote the product topology on $X \times X$. Define the diagonal $\Delta \subset X \times X$ by $\Delta = \{(x, x) \mid x \in X\}$. Show that $(X, \mathcal{T})$ is Hausdorff if and only if Δ is a closed subset in $(X \times X, \mathcal{U})$.